HW 5 - 8

1. increasing: $(-\infty, 1)$
   decreasing: $(1, \infty)$
   rel min: none
   rel max: (1, 3)

2. increasing: (-3, 1)
   decreasing: $(-\infty, -3)$, $(1, \infty)$
   rel min: (-3, -4)
   rel max: (1, 4)

3. look left to right where you would "climb the hill", graph goes higher

4. a point on the graph higher that those on either side of it

5. determine if the leading coefficient of the polynomial is + or - and decide if the degree is odd or even

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6. Graph see next page
   increasing: (-1.44, 0), $(.69, \infty)$
   decreasing: $(-\infty, -1.44)$, (0, .69)
   rel min: (-1.44, -2.83), (.69, -.40)
   rel max: (0, 0)

7. Graph see next page
   increasing: (-1.79, 1.12)
   decreasing: $(-\infty, -1.79)$, $(1.12, \infty)$
   rel min: (-1.79, -8.21)
   rel max: (1.12, 4.06)

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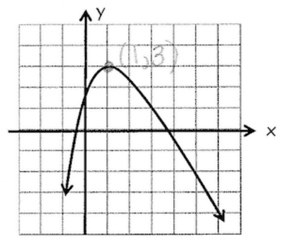
For each of the following, determine the intervals on which the graph is increasing and decreasing. Find all relative minima and maxima.

1.



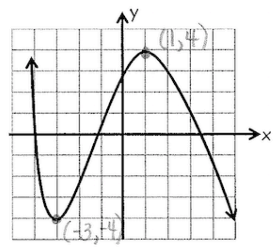
Increasing: $(-\infty, 1)$

Decreasing: $(1, \infty)$

Rel Min: none

Rel Max: $(1, 3)$

2.



Increasing: $(-3, 1)$

Decreasing: $(-\infty, -3), (1, \infty)$

Rel Min: $(-3, -4)$

Rel Max: $(1, 4)$

3. How do you determine where a graph is increasing?
look left to right where you would "climb the hill" - graph goes higher

4. In your own words, what is a relative minimum?
a place on the graph higher than those on either side of it

5. How do you determine the end behavior of the graph of a polynomial function?
Determine if the leading coefficient of the polynomial is + or − and decide if the degree is odd or even

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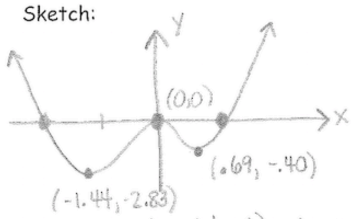
In 6 & 7, state the degree of the polynomial, find the zeros of each polynomial, state the multiplicity of each. Sketch. Using your calculator, determine relative min/max and where it's increasing/decreasing.

6. $P(x) = x^2(x + 2)(x - 1)$

Degree: 4

Z	M	T/C
-2	1	C
0	2	T
1	1	C

Sketch:



Increasing: $(-1.44, 0), (.69, \infty)$

Decreasing: $(-\infty, -1.44), (0, .69)$

Rel Min: $(-1.44, -2.83), (.69, -.40)$

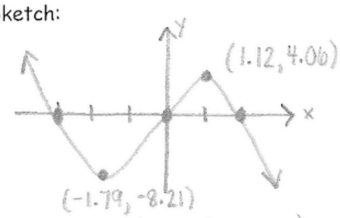
Rel Max: $(0, 0)$

7. $Q(x) = -x(x + 3)(x - 2)$

Degree: 3

Z	M	T/C
-3	1	C
0	1	C
2	1	C

Sketch:



Increasing: $(-1.79, 1.12)$

Decreasing: $(-\infty, -1.79), (1.12, \infty)$

Rel Min: $(-1.79, -8.21)$

Rel Max: $(1.12, 4.06)$

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Day 9 Remainder Theorem

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Warm-Up: Each of you will be assigned one of these three problems.

Remember: $F(x) \div G(x) = Q(x)$ with a remainder of $R(x)$

Which is easier to read as:

work space below - use for your problem...

Dividend → $F(x) = G(x) \cdot Q(x) + R(x)$

Divisor Quotient Remainder

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1. Consider the polynomial function $f(x) = 3x^2 + 8x - 4$

a. Using long division, divide f(x) by x - 2

$Q(x) = 3x+14$

$R(x) = 24$

So $f(x) = (x-2)(3x+14)+24$

$$\begin{array}{r} 3x + 14 \\ x-2\,\overline{)\,3x^2 + 8x - 4} \\ -3x^2 + 6x \\ \hline 14x - 4 \\ -14x + 28 \\ \hline 24 \end{array}$$

Aside:

$3x(x-2) = 3x^2 - 6x$

$14(x-2) = 14x - 28$

b. Find f(2)

$f(2) = 3(2)^2 + 8(2) - 4$

$= 12 + 16 - 4$

$= 24$

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2. Consider the polynomial function $g(x) = x^3 - 3x^2 + 6x + 8$

a. Using long division, divide g(x) by x + 1

$Q(x) = x^2 - 4x + 10$

$R(x) = -2$

So $g(x) = (x+1)(x^2 - 4x + 10) - 2$

$$\begin{array}{r} x^2 - 4x + 10 \\ x+1\,\overline{)\,x^3 - 3x^2 + 6x + 8} \\ -x^3 - x^2 \\ \hline -4x^2 + 6x \\ 4x^2 + 4x \\ \hline 10x + 8 \\ -10x - 10 \\ \hline -2 \end{array}$$

Aside:

$x^2(x+1) = x^3 + x^2$

$-4x(x+1) = -4x^2 - 4x$

$10(x+1) = 10x + 10$

b. Find g(-1)

$g(-1) = (-1)^3 - 3(-1)^2 + 6(-1) + 8$

$g(-1) = -1 - 3 - 6 + 8$

$g(-1) = -2$

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3. Consider the polynomial function $h(x) = x^3 + 0x^2 + 2x - 3$

a. Using long division, divide $h(x)$ by $x - 3$

$Q(x) = x^2 + 3x + 11$

$R(x) = 30$

So $h(x) = (x-3)(x^2+3x+11) + 30$

$$
\begin{array}{r}
x^2 + 3x + 11 \\
x-3 \,\big)\, x^3 + 0x^2 + 2x - 3 \\
-x^3 + 3x^2 \\
\hline
3x^2 + 2x \\
-3x^2 + 9x \\
\hline
11x - 3 \\
-11x + 33 \\
\hline
30
\end{array}
$$

b. Find $h(3)$

$h(3) = (3)^3 + 2(3) - 3$

$h(3) = 27 + 6 - 3$

$h(3) = 30$

Aside:

$x^2(x-3) = x^3 - 3x^2$

$3x(x-3) = 3x^2 - 9x$

$11(x-3) = 11x - 33$

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Write in the answers for all parts, gathered from the class discussion. What pattern do you see? (Answers will be posted - get them from your teachers website)

The remainder is the same as the function value of the possible zero.

Look at #1: Possible zero: 2

$R(2) = f(2)$

What can we say about the connection between dividing a polynomial, P, by x – a and the value of P(a)?

Remainder = P(a)

$R(x) = P(a)$

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Day 9 Remainder Theorem Day 2

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In algebra, the **remainder theorem** is an application of polynomial long division.

The remainder of a polynomial p(x) divided by a linear divisor (x - c) is equal to p(c).

What does that mean?

If you divide a polynomial P(x) by a possible factor (x - c), you will get a remainder that is equal to the function value of the corresponding possible zero.

(x-2)

P(2)= remainder

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~~Formally~~: $P(x) = q(x)(x - a) + P(a)$ Remainder
Formula

remainder $= 0 = P(a)$

Why Is This Useful?

Knowing that x - c is a factor is the same as knowing that c is a root (and vice versa).

The **factor** "x - c" and the **root** "c" are the same thing!
Now try these: Use the remainder theorem to determine the remainder.
Rem $= P(c)$

1. $(-x^3 + 6x - 7) \div (x - 2)$ $x = 2$

$P(2) = -(2)^3 + 6(2) - 7$
$P(2) = -8 + 12 - 7$
$P(2) = -3$
$\therefore R(x) = -3$

2. $(x^3 + x^2 - 5x - 6) \div (x + 2)$ $x = -2$

$P(-2) = (-2)^3 + (-2)^2 - 5(-2) - 6$
$P(-2) = -8 + 4 + 10 - 6$
$P(-2) = -4 + 4$
$P(-2) = 0$
$\therefore R(x) = 0$

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What do you think it means if the remainder is 0?
That the divisor is a factor of the polynomial
and the x-value is a zero of the polynomial.
(x-intercept, root)

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What do you think it means if the remainder is 0?

The divisor is a factor of the polynomial; the corresponding x-value is a zero of the polynomial.

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Another use for the remainder theorem: you can determine very quickly if a given binomial is a factor of a polynomial without doing long division or factoring.

Using the remainder theorem, determine if the given binomial is a factor of the given polynomial. If so, remainder is zero

1. $(x^3 - x^2 - x - 2) \div (x - 2)$ $x=2$

$P(2) = 2^3 - 2^2 - 2 - 2 = 8 - 4 - 4 = 0$ remainder $\therefore$ yes, $x-2$ is a factor.

2. $(x^4 - 8x^3 - x^2 + 62x - 34) \div (x - 7)$ $x=7$

$P(7) = 7^4 - 8(7)^3 - 7^2 + 62(7) - 34 = 8 \neq 0$

$\therefore$ no, $x-7$ is not a factor.

and 7 is not a zero of the function

(7, 8)

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3. Given the polynomial $P(x) = x^3 + kx^2 + x + 6$

a. Find the value of k so that x + 1 is a factor of P. $P(-1)=0$

$-1 \to 0$
$x \to y$

$P(-1)=0$
$0=(-1)^3+k(-1)^2+(-1)+6$
$0=-1+k+5$
$0=4+k$
$k=-4$

$P(x)=x^3-4x^2+x+6$

b. Find the other two factors of P for the value of k found in part a.

$$\begin{array}{r} x^2-5x+6 \\ x+1\,\overline{)\,x^3-4x^2+x+6} \\ -(x^3+x^2) \\ \hline -5x^2+x \\ -(-5x^2-5x) \\ \hline 6x+6 \\ -(6x+6) \\ \hline 0 \end{array}$$

$P(x)=(x+1)(x^2-5x+6)$
$P(x)=(x+1)(x-3)(x-2)$

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