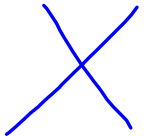


Solving Systems of Equations Algebraically

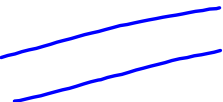
Consistent → a system of equations that has at least one solution

Inconsistent → a system that has no solutions

If a system of two linear equations in two variables has an infinite number of solutions, the equations are **dependent**. Otherwise, they are **independent**.



Consistent
Independent



Inconsistent
independent



Consistent
dependent

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Solve. Classify as consistent or inconsistent. Dependent or independent.

Solve the following systems by substitution.

1. $2x - y = -7$
 $x = 2y - 8$
 $2(2y - 8) - y = -7$
 $4y - 16 - y = -7$
 $3y = 9$
 $y = 3$
 $x = 2(3) - 8$
 $x = -2$ $(-2, 3)$
 Consistent
independent

2. $x - 2y = 5$
 $2x + y = 3$ $y = -2x + 3$
 $x - 2(-2x + 3) = 5$
 $x + 4x - 6 = 5$
 $5x - 6 = 5$
 $5x = 11$
 $x = 11/5$
 $11/5 - 2y = 5$
 $-2y = 5 - 11/5$
 $-2y = 14/5$
 $-2y = 14/5$
 $y = 14/5 \cdot -1/2 = -7/5$
 $(\frac{11}{5}, -\frac{7}{5})$
 Consistent
independent

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Solve the following systems by elimination.

3. $3x - 2y = 4$

$$\begin{array}{r}
 (2x + y = -1) \cdot 2 \\
 \hline
 3x - 2y = 4 \\
 4x + 2y = -2 \\
 \hline
 7x = 2 \\
 x = \frac{2}{7} \\
 \hline
 2(\frac{2}{7}) + y = -1 \\
 \frac{4}{7} + y = -1 \quad (\frac{2}{7}, \frac{-11}{7}) \\
 y = -1 - \frac{4}{7} \\
 y = -\frac{7}{7} - \frac{4}{7} = -\frac{11}{7} \\
 \hline
 \text{Consistent} \\
 \text{independent}
 \end{array}$$

$$\begin{array}{r}
 4. \left(\frac{x}{3} - \frac{y}{2} = -3 \right) \cdot 6 \\
 \left(\frac{2x}{5} + \frac{y}{5} = -2 \right) \cdot 5 \\
 \hline
 2x - 3y = -18 \\
 (2x + y = -10) \cdot -1 \\
 \hline
 2x - 3y = -18 \\
 -2x - y = 10 \\
 \hline
 -4y = -8 \\
 y = 2 \\
 \hline
 \frac{x}{3} - \frac{2}{2} = -3 \\
 \frac{x}{3} = -3 + 1 \\
 \frac{x}{3} = -2 \rightarrow x = -6 \\
 \hline
 (-6, 2) \\
 \text{Consistent} \\
 \text{independent}
 \end{array}$$

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$$\begin{array}{r}
 5. \left(2x - 8y = 2 \right) \cdot 3 \\
 \left(3x - 12y = 3 \right) \cdot -2 \\
 \hline
 6x - 24y = 6 \\
 -6x + 24y = -6 \\
 \hline
 0 = 0
 \end{array}$$

True.
 $\rightarrow$ Infinite Solutions
 Consistent
 dependent

$$\begin{array}{r}
 6. \left(2x + 5y = 8 \right) \cdot 3 \\
 \hline
 6x + 15y = 18 \\
 -6x - 15y = -24 \\
 \hline
 6x + 15y = 18 \\
 0 = -6
 \end{array}$$

False - contradiction
 $\emptyset$ or $\{ \}$
 inconsistent
 independent

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$$\begin{aligned} 7. \textcircled{1} \quad x + 2y + 3z &= -2 \\ \textcircled{2} \quad 2x + 6y + z &= 2 \\ \textcircled{3} \quad 3x + 3y + 10z &= -2 \end{aligned}$$

St.1 Elim x for $\textcircled{1}$ and $\textcircled{2}$

$$\begin{aligned} -2\textcircled{1} + \textcircled{2} \\ -2x - 4y - 6z &= 4 \\ 2x + 6y + z &= 2 \end{aligned}$$

$$2y - 5z = 6 \quad \textcircled{4}$$

St.2 Elim x for $\textcircled{1}$ and $\textcircled{3}$

$$\begin{aligned} -3\textcircled{1} + \textcircled{3} \\ -3x - 6y - 9z &= 6 \\ 3x + 3y + 10z &= -2 \end{aligned}$$

$$-3y + z = 4 \quad \textcircled{5}$$

St.3 Solve for y and z (elim)

$$\begin{aligned} 2y - 5z &= 6 \\ (-3y + z = 4) \cdot 5 \end{aligned}$$

$$\begin{aligned} 2y - 5z &= 6 \\ -15y + 5z &= 20 \end{aligned}$$

$$-13y = 26$$

$$y = -2$$

$$-3(-2) + z = 4$$

$$6 + z = 4$$

$$z = -2$$

St.4 $x =$ _____

$$\textcircled{1} \quad x + 2(-2) + 3(-2) = -2$$

$$x - 4 - 6 = -2$$

$$x = 8$$

$$\text{or } (8, -2, -2)$$

$$\begin{aligned} x &= 8 \\ y &= -2 \\ z &= -2 \end{aligned}$$

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Do the following as part of your homework: $3x - 2y + z = -3$

$$-x + y - 2z = -5$$

$$x + y + z = 5$$

Homework: PP. 683 - 684
#17, 24, 25, 27, 32, 34, 36, 39
and the problem above

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