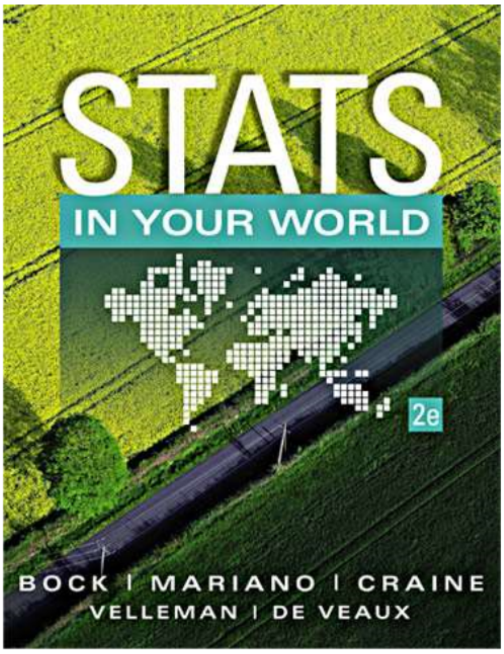
Chapter 8

What's My Curve?

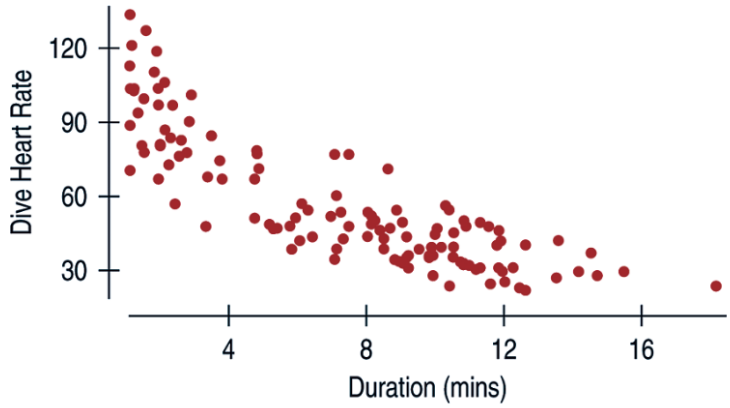


Straight to the Point

- We cannot use a linear model unless the relationship between the two variables is linear.

Straight to the Point (cont.)

- The relationship between *Dive Heart Rate* (in beats per minute) and *Duration* (in minutes) looks fairly linear at first:



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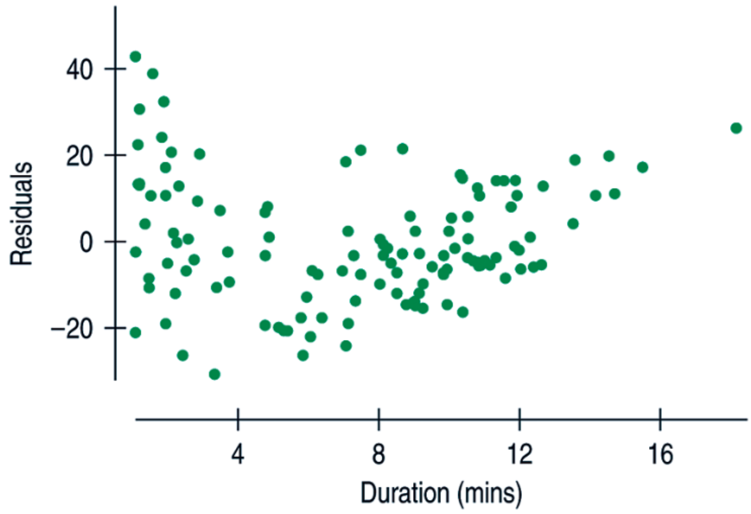
Straight to the Point (cont.)

- A look at the residuals plot shows a problem:



Getting the "Bends"

- If there's a clear bend in the residuals, your model has missed something important about the relationship, and it's time to look for a better model.
- We will explore two different kinds of curves, although this topic is complex and there are many, many options.
- But hopefully our residual plots will show… nothing. Which indicates our model is strong.

Exponential Models

- Populations tend to grow exponentially.
- So does money when inflation increases costs by a given percentage year after year.
- Other things *shrink* exponentially. For example, the way your body metabolizes a certain percentage of a drug may decay exponentially.
- Radioactive decay is another example of an exponential model.

Exponential Models (cont.)

- Equations of exponential models involve (surprise!) exponents, and look like this:

$$\hat{y} = a\left(b^{x}\right)$$

Exponential Models (cont.)

- We see that *a* represents the model's starting value.
- The value *b* represents the growth rate (or decay rate).
- If $b = 1.02$, that's 102%. A 2% growth rate.
- If the model is decreasing 15% for every 1 unit of change in *x*, then the model's value of *b* would be 100% - 15% = 0.85.

$y = 4(1.23)^x$

starting value: 4

increasing/decreasing

by 23 %

1.23 = 123%

$y = .75(1.15)^x$

starting value: .75

increasing/decreasing

by 15 %

1.15 = 115%

$y = 7(0.85)^x$

starting value: 7

increasing/decreasing

by 15 %

.85 = 85%

100% - 85%

15% ↓

$y = .25(\underline{0.7})^x$

starting value: .25

increasing/decreasing

by 30 %

.7 = 70%

100 - 70

$y = 4000(3.2)^x$

starting value: 4000

increasing/decreasing

by 220 %

$3.2 = 320\%$
-100%
220%

$y = 300(.5)^x$

starting value: 300

increasing/decreasing

by 50 %

$.5 = 50\%$

$y = 7(4)^x$

starting value: 7

increasing/decreasing

by 300 %

$4 = 400\%$
-100%
300%

$y = 7{,}800(\frac{1}{4})^x$

starting value: 7800

increasing/decreasing

by 75 %

$\frac{1}{4} = .25$
25%
$100 - 25 = 75\%$

<u>Linear models:</u>

as x increases, we add/subtract a set amount

Ex: $y = 6 + 3x$ (as x increase by 1, y increases by 3)

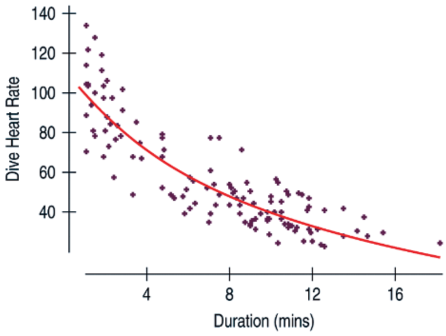
<u>Exponential Models:</u>

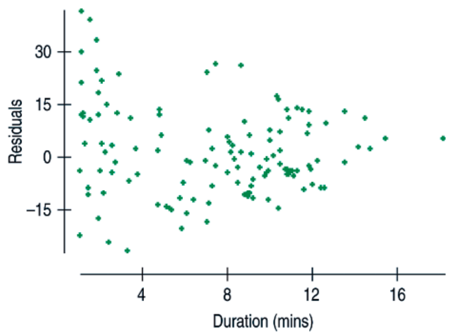
as x increases, we multiply/divide by a set amount

Ex: $y = 6(3)^x$ (as x increase by 1, y increases by 3 times)

A Model for Penguin Dives

- The researchers studying emperor penguins set out to understand how the duration of a dive is related to the penguin's heart rate.
- Here are the scatterplot showing the curve and the resulting residuals plot:





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A Model for Penguin Dives (cont.)

- That's clearly an improvement.
- The residuals now look random.
- The equation for our exponential model is:

$$\widehat{DiveHeartRate} = 102.32\left(0.91\right)^{Duration}$$

A Model for Penguin Dives (cont.)

- The value of *a* tells us our model estimates that penguins' heart rates should average around 102.32 beats per minute when they're not diving.
- The value of *b* indicates that heart rates tend to decrease exponentially about 1.00 - 0.91 = 9% for each minute a dive lasts.

Homework:

Worksheet: Linear and Exponential Patterns

WS# 8-5

$43(2)^8$

1st