Homework Answers:

1. **Bolts.** Choice A. $\widehat{fail} = 10.965(1.003)^{load} = 10.965(1.003)^{600} \approx 66.2$

2. **Exponential models.** Choice C. The growth factor of 1.23 represents 123% *of* the previous value of x, which means 100% plus an additional 23%. That's the growth.

5. **Marriage 2010.** Choice D.

6. **Identify, part I.**

   a) Linear. The y variable increases by 150 each time the x variable increases by 2.

   b) Exponential. The y variable doubles each time the x variable increases by 2.

8. **Identify, part III.**

   a) Exponential. The y variable is multiplied by 2/3 each time the x variable increases by 10.

   b) Linear. The y variable decreases by 15 each time the x variable increases by 10.

12. Bacteria.

a) The model predicts that the lab started with 500 bacteria. $\widehat{Pop} = 500(1.06^{Day})$

b) The growth rate according to the model was 6% per day. $\widehat{Pop} = 500(1.06^{7})$

c) The lab should expect about 752 bacteria in a week. $\widehat{Pop} \approx 752$

13. Homes.

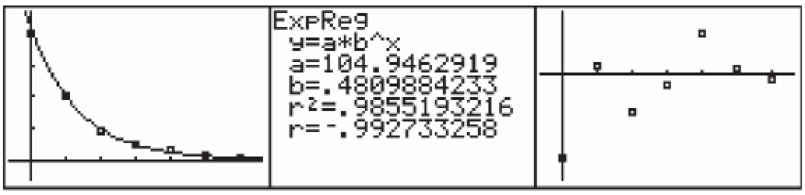
a) The model predicts that there were 350 homes in 1980. $\widehat{Homes} = 350(1.08^{year})$

b) The growth rate according to the model was 8% per year. $\widehat{Homes} = 350(1.08^{40})$

c) The model predicts about 7604 homes in 2020. $\widehat{Homes} \approx 7604$

24. Dying coins.

a) The scatterplot, exponential regression output and residuals plot at the right show the exponential model,



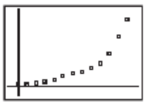
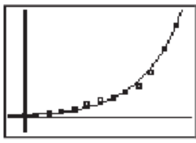
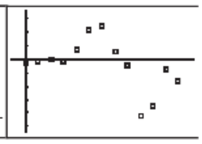
$\widehat{Coins} = 104.95(0.48^{Toss})$. This model is appropriate, since the residuals look reasonably random.

b) The growth rate, 0.48, represents a 52% "death" rate for the coins. This is reasonably close to the theoretical death rate of 50%.

Statistics Chapter 8: Step by Step Exponential Models – KEY

Here is a record of annual world crude oil production from 1900 to 1960 in five-year increments between 1900 and 1960. What model describes the growth in production over this time frame?

L1 x=	0	5	10	15	20	25	30	35	40	45	50	55	60
Year	1900	1905	1910	1915	1920	1925	1930	1935	1940	1945	1950	1955	1960
Millions of barrels	149	215	328	432	689	1069	1412	1655	2150	2595	3803	5626	7674

Think	• State the problem. • Identify the variables and report the W's. • Make a picture. • Check the appropriate assumptions and conditions.	We want to develop a model that predicts crude oil production based on the year. Let *years* be the number of years since 1900, and let *oil* represent the number of millions of barrels produced. We have information about world oil production every 5 years from 1900 to 1960. Both variables are quantitative, and the scatterplot shows a relationship that is strong, positive, and curved. We should try an exponential model. 
Show	• Justify the use of an exponential model. • Check the residuals. • Give the equation.	The exponential model seems to fit the scatter plot well. The residuals plot shows some pattern, but the residuals are small. This certainly fits better than a linear model would. The exponential model is $\widehat{Oil} = 172.68(1.066)^{Years}$  
Tell	• Interpret your model in the context of the problem.	According to the model, world oil production was about 172.68 million barrels in 1900 (we can see from our data that is was actually 149 million barrels, but remember that our model is a prediction). Furthermore, the model predicts that world oil production increased by about 6.6% per year from 1900 to 1960.

Plan B: Power Models

$\hat{y} = a(b)^x$

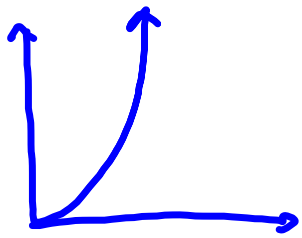
- Not all curves are exponential.
- Sometimes a power model is useful.
- Equations of **power models** look like this:

$$\hat{y} = a\left(x^b\right)$$

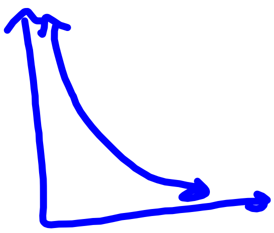
- This time we are raising *x* to a power.

Let's graph the following Power functions so we can see what they look like. Set window to x's and y's from 0 to 10.

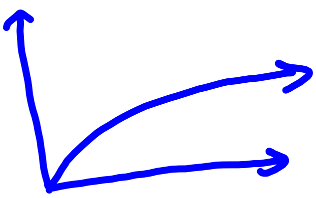
$y = 2(x)^2$

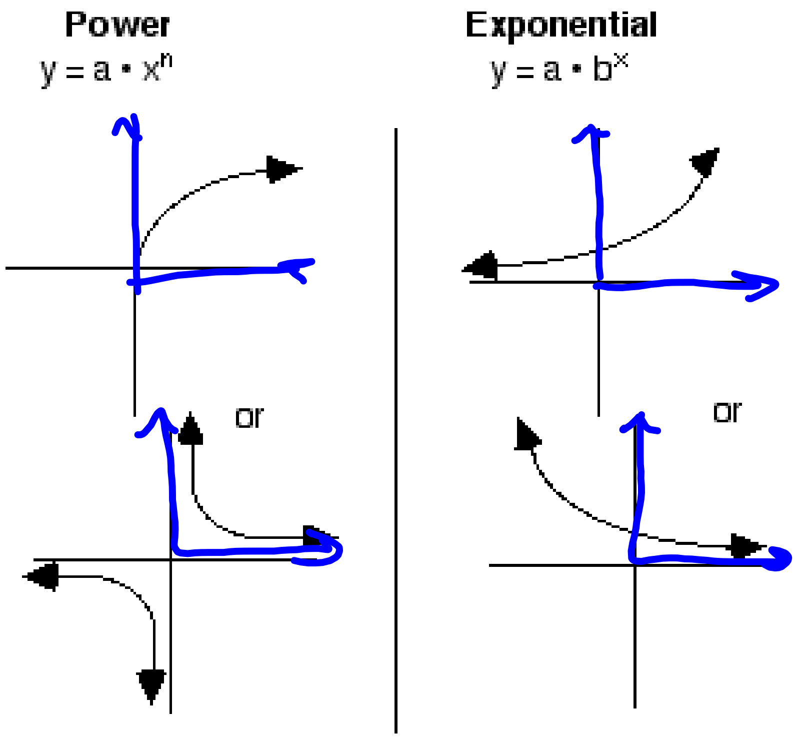


$y = 2(x)^{-2}$



$y = 2(x)^{\frac{1}{2}}$

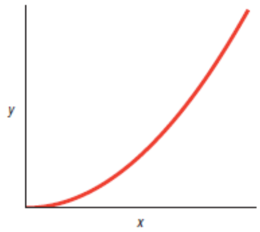




We are generally looking at 1st Quadrant data only for real-world applications.

Equations of **power models** look like this:

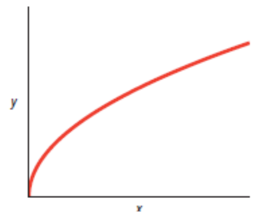
$$\hat{y} = a(x^b)$$



You have seen some common examples in your algebra classes:

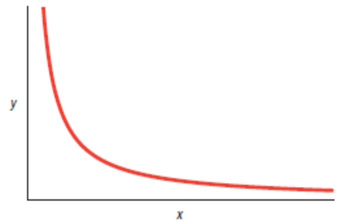
- $\hat{y} = ax^2$

  We square the x value (sometimes called a quadratic model).



- $y = ax^{1/2}$

The exponent of ½ means take the square root of the x value (sometimes written $y = a\sqrt{x}$).



- $y = ax^{-1}$

The exponent of -1 tells us to use the reciprocal of the x value (sometimes written $y = \frac{a}{x}$).

Do The Math

Working with power functions involves understanding all kinds of exponents. Let's review:

- Negative exponents represent reciprocals: $4^{-1} = \frac{1}{4}$; $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
- Fractional exponents represent roots: $25^{\frac{1}{2}} = \sqrt{25} = 5$; $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

5. Without using your calculator, evaluate:

a) $5^{-2} = \frac{1}{5^2} = 1/25$
b) $16^{\frac{1}{2}} = \sqrt{16} = 4$
c) $16^{-\frac{1}{2}} = \frac{1}{\sqrt{16}} = 1/4$
d) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$
e) $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$
f) $27^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{27}} = 1/3$

6. Make a table of values and sketch the graph of each power function $y = ab^x$ below. Describe the pattern of change of each using one of the following:
- as x increases, y decreases at a decreasing rate
- as x increases, y decreases at an increasing rate
- as x increases, y increases at a decreasing rate
- as x increases, y increases at an increasing rate

a) $y = 0.5x^2$, using $x = 0, 1, 2, 3, 4, 5, 6$
b) $y = 40x^{-1}$, using $x = 1, 2, 4, 5, 8, 10$
c) $y = 2x^{0.5}$, using $x = 0, 1, 4, 9, 16, 25$
d) $y = 30x^{-0.5}$, using $x = 1, 4, 9, 16, 25, 36$

(Check your answers on page 217.)

6. a) As x increases, y increases at an increasing rate

x	0	1	2	3	4	5	6
y	0	0.5	2	4.5	8	12.5	18

b) As x increases, y decreases at a decreasing rate

x	1	2	4	5	8	10
y	40	20	10	8	5	4

c) As x increases, y increases at a decreasing rate

x	0	1	4	9	16	25
y	0	2	4	6	8	10

d) As x increases, y decreases at a decreasing rate

x	1	4	9	16	25	36
y	30	15	10	7.5	6	5

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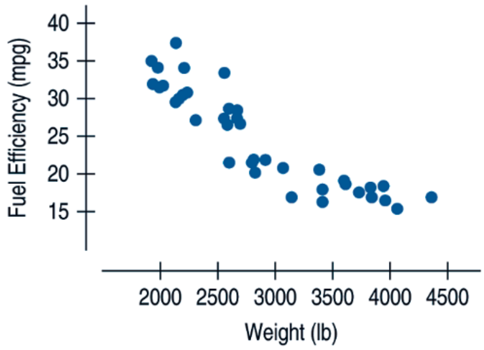
$y = \frac{5}{x}$

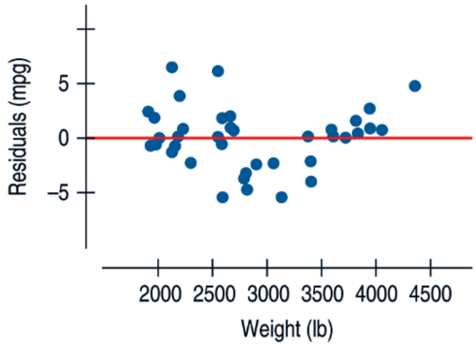
Power models are useful in many situations. Rates—miles per hour, say—may use a reciprocal model; think minutes per mile. A variable based on area—for instance, the amount of paint needed—may involve squaring or square roots. Similarly, quantities based on volume—such as weight—may involve cubes or cube roots. If you have studied Physics you learned that energy variables like the intensity of light or force of gravity follow inverse square laws; those are power models using the exponent −2.

If you're wondering how you'll know what exponent to try, we have good news for you: Your calculator or a computer can figure that out. Your jobs are to think of trying a power model, and then to use the residuals plot to be sure that model is appropriate.

Modeling Fuel Economy

- This scatterplot shows the *Weight* (in pounds) and *Fuel Efficiency* (in mpg) for 38 cars.
- Note that the residual plot for a linear model has a shape that is clearly bent.





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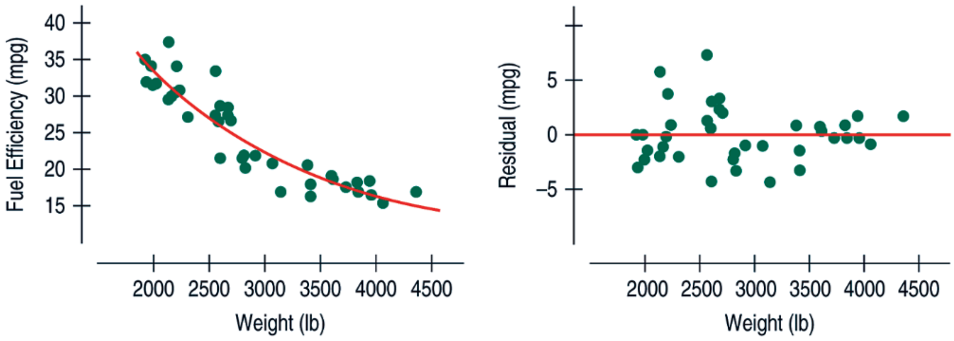
Modeling Fuel Economy (cont.)

- We have eliminated the linear model.
- Because fuel economy is a ratio and reciprocals can be expressed using the exponent of –1, we try a power model:



Modeling Fuel Economy (cont.)

- Not an ideal residual plot, but models are rarely perfect!
- This power model should be more useful:

$$\widehat{MPG} = 79157\left(Weight\right)^{-1.022}$$

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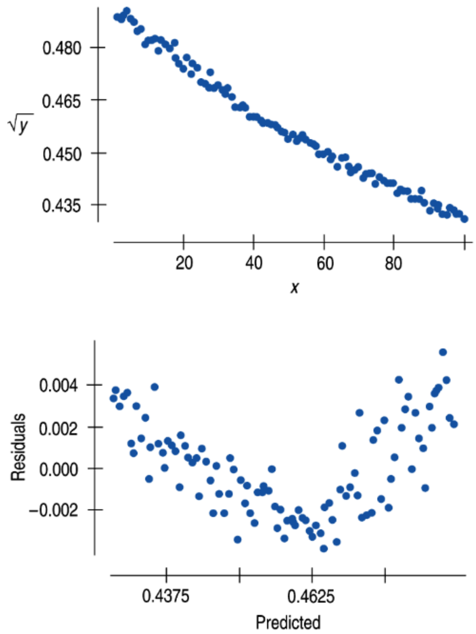
In Search of the Perfect Model – Not!

- The real world is messy.
- Some models are very useful.
- Rarely are models perfect.
- A wise person is suspicious of data that fits a model too closely!

 Chapter 8, Slide 16

What Can Go Wrong?

- Remember the basic rule of data analysis: Make a Picture!
- Don't expect your model to be perfect.



What Can Go Wrong? (cont).

- Watch out for scatterplots that turn around. You won't learn in this class how to create models that deal with this.
- Don't round too much! When dealing with exponents, even minor changes can make big differences.

Just Checking

After reading each of the following descriptions, tell whether you'd try a linear model, an exponential model, or a power model. If you think none of these might be useful, explain why.

1. You want to model the growth of a college savings fund that Uncle Rich set up when you were born. It has been sitting in the bank earning compound interest ever since.
2. You want to model the relationship between prices for common items like food and clothing in Boston and Tokyo. Your scatterplot shows a generally straight pattern with only moderate amounts of scatter.
3. You want to model the average weekly temperature in Denver over a year.
4. You want to model the relationship between the *Diameter* of rivets used to hold metal beams together and the *Strength* of those rivets (the weight they can hold without breaking).

1. Exponential (Interest earned each year is a percentage of the amount present.)
2. Linear
3. None of these. The temperature probably starts low in the winter months, goes up during the spring, peaks in the summer, then goes back down again through the fall and into the next winter.
4. Power. (Strength may depend on the cross-sectional area of the rivet.)

No Homework