The General Multiplication Rule

- When two events **A** and **B** are independent, we can use the multiplication rule for independent events from Chapter 14:

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$$

- However, when our events are not independent, this earlier multiplication rule does not work. Thus, we need the General Multiplication Rule.

The General Multiplication Rule (cont.)

- We encountered the general multiplication rule in the form of conditional probability.
- Rearranging the equation in the definition for conditional probability, we get the General Multiplication Rule:
  - For any two events **A** and **B**,

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B} \mid \mathbf{A})$$

or

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{B}) \times P(\mathbf{A} \mid \mathbf{B})$$

Two cards are drawn at random without replacement.

What is the probability they are both aces?

The two draws are not independent.

P(ace,ace) = P(ace)*P(ace|1st draw is ace)

$4/52 \times 3/51 = \frac{12}{2652}$

With Replacement → independent

$P(Ace, Ace) = P(Ace) \times P(Ace)$

$= 4/52 \times 4/52 = 16/2704$

Depending on Independence

- It's much easier to think about independent events than to deal with conditional probabilities.
  - It seems that most people's natural intuition for probabilities breaks down when it comes to conditional probabilities.
- Don't fall into this trap: whenever you see probabilities multiplied together, stop and ask whether you think they are really independent.

Drawing Without Replacement

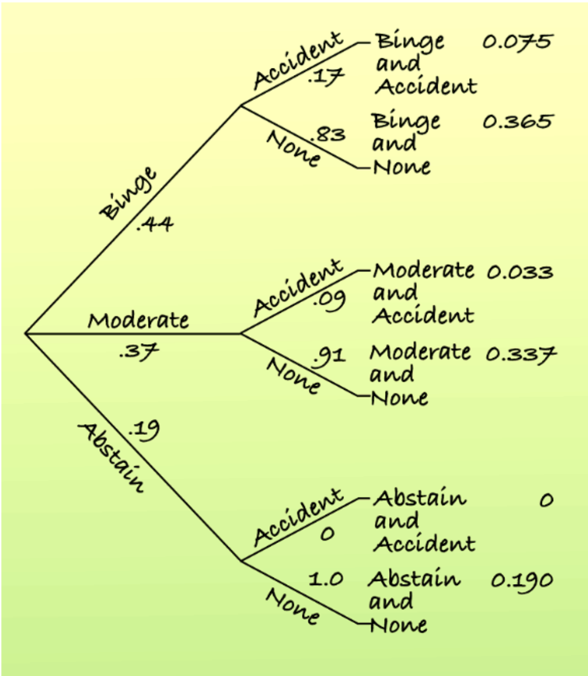
- Sampling without replacement means that once one individual is drawn it doesn't go back into the pool.
  - We often sample without replacement, which doesn't matter too much when we are dealing with a large population.
  - However, when drawing from a small population, we need to take note and adjust probabilities accordingly.
- Drawing without replacement is just another instance of working with conditional probabilities.

Tree Diagrams

- A tree diagram helps us think through conditional probabilities by showing sequences of events as paths that look like branches of a tree.
- Making a tree diagram for situations with conditional probabilities is consistent with our "make a picture" mantra.

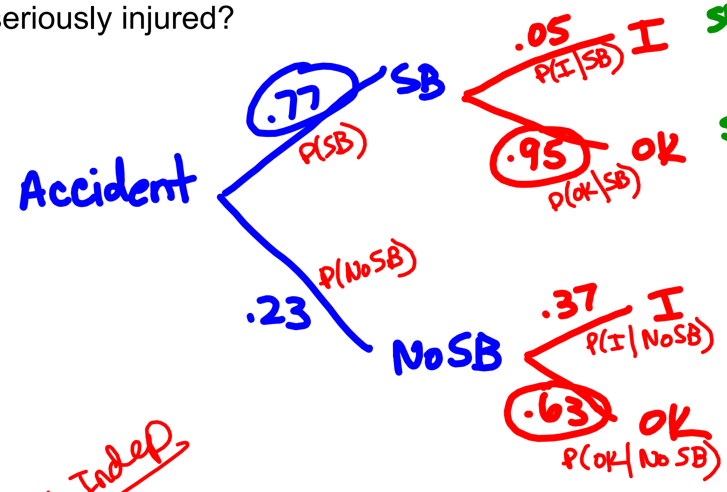
Tree Diagrams (cont.)

- This figure is a nice example of a tree diagram and shows how we multiply the probabilities of the branches together.
- All the final outcomes are disjoint and must add up to one.
- We can add the final probabilities to find probabilities of compound events.



Maryland Highway Safety Council found recently that in 77% of all accidents, the driver was wearing a seatbelt (SB). Accident reports suggested that 95% of those drivers escaped serious injury(I), but only 63% of non-belted drivers were so fortunate.

Overall, what's the probability that a driver involved in an accident in Maryland was seriously injured?



$SB \cap I$: $.77 \times .05 = .0385$

$SB \cap OK$: $.77 \times .95 = .7315$

$NoSB \cap I$: $.23 \times .37 = .0851$

$NoSB \cap OK$: $.23 \times .63 = .1449$

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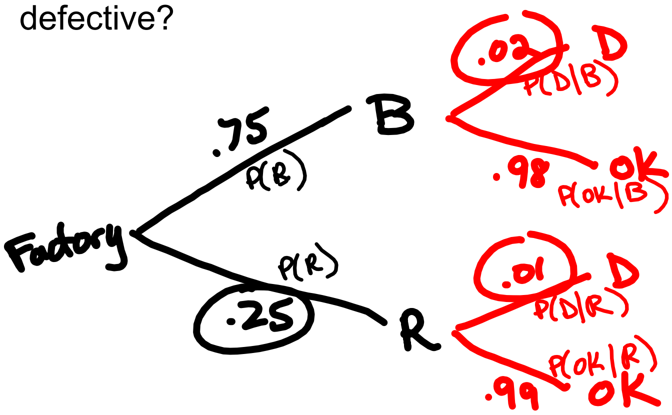
Not Indep.

$$P(I) = P(SB \cap I) + P(NoSB \cap I)$$

$$.0385 + .0851 = .1236$$

A battery factory produces both regular batteries (B) and rechargeable batteries (R). 25% of the batteries produced at the factory are rechargeable. Past history shows that 2% of regular batteries are defective and 1% of rechargeable batteries are defective.

What's the probability that a battery chosen at random from this factory will be defective?



$= P(B \cap D) = .75 \times .02 = .0150$

$= P(B \cap OK) = .75 \times .98 = .7350$

$= P(R \cap D) = .25 \times .01 = .0025$

$= P(R \cap OK) = .25 \times .99 = .2475$

$$P(D) = .0150 + .0025 = .0175$$

Homework

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P(MakesEt) = .695

P(Grad) = .795