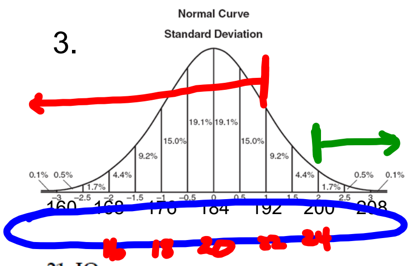
Do Warm-Up on Pg. 7 while I check Homework

3.



5a. 68% (within 1 SD)

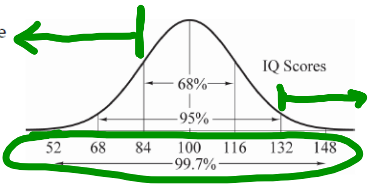
b. 84% (50%+34%)

c. 2.5% (above 2 SD above the mean)

2.3

21. IQ.

The Normal model for IQ scores is at the right.



23. IQs again.

a) Approximately 95% of the IQ scores are expected to be within the interval 68 to 132 IQ points.

b) Approximately ~~84%~~ 16% of IQ scores are expected to be ~~above~~ below 84 IQ points.

c) Approximately 2.5% of the IQ scores are expected to be above 132.

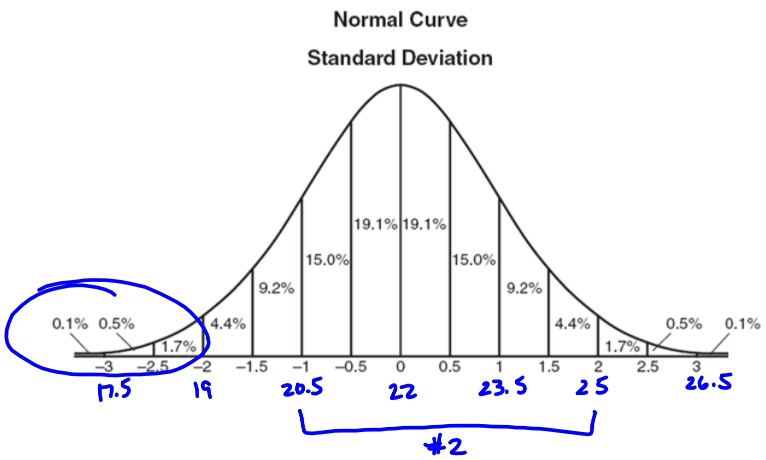
25. Genius.

Any IQ more than 2 standard deviations above the mean, or more than 100 + 2 (16) = 132 might be considered unusually high. We would expect to find someone with an IQ over 100 + 3(16) = 148 very rarely.

Z - Scores

Warm-Ups:

1. The average gas mileage for a small SUV is 22 mpg with a standard deviation of 1.5. Label the gas mileages along the bottom of the normal curve.



2. What percent of small SUV's get between 20.5 and 25 mpg?

$15 + 19.1 + 19.1 + 15 + 9.2 + 4.4 = 81.8\%$

3. If there are 25,000 small SUV's in NY state, how many of them get between 20.5 and 25 mpg?

$.818(25{,}000) = 20{,}450$ SUV's

Standard Deviation as a Ruler:

- If we want to determine how unusual a piece of data is, it is best to see how far the data is away from the mean.
- We measure this distance in terms of how many standard deviations the piece of data is away from the mean. The more unusual the piece of data is, the more standard deviations it will be from the mean (above or below).
- We can use this to compare any one piece of data (individual) to the whole set of data (group).

Standardizing with *z*-scores

- We compare individual data values to their mean, relative to their standard deviation using the following formula:

$$z = \frac{(y - \bar{y})}{s} = \frac{\text{Value} - \text{mean}}{\text{Std. dev}}$$

y = one data

$\bar{y}$ = mean

s = std. dev.

- We call the resulting values standardized values, denoted as *z*. They can also be called *z*-scores.

z-score = # on std. dev. from the mean

Standardizing with *z*-scores (cont.)

- Standardized values have no units.
- *z*-scores measure the distance of each data value from the mean in standard deviations.
- A negative *z*-score tells us that the data value is *below* the mean, while a positive *z*-score tells us that the data value is *above* the mean.
  - A *z*-score of 2 says that a data value is 2 standard deviations above the mean.
  - A *z*-score of –1.6 means a data value is 1.6 standard deviations below the mean.

 Chapter 5, Slide 9

Benefits of Standardizing

- Standardized values have been converted from their original units to the standard statistical unit of *standard deviations from the mean*.
- Thus, we can compare values that are measured on different scales, with different units, or from different populations.

 Chapter 5, Slide 10

Keeping your signs straight

- Remember that a negative *z*-score tells us that the data value is *below* the mean, while a positive *z*-score tells us that the data value is *above* the mean.
- Pro-tip: if you forget which order to subtract, use common sense! Above or below? Positive or negative?

 Chapter 5, Slide 16

z-scores & the Normal model

- There is no universal standard for *z*-scores, but there is a model that shows up over and over in Statistics.
- This model is called the Normal model (You may have heard of “bell-shaped curves.”).
- Normal models are appropriate for distributions whose shapes are unimodal and roughly symmetric.
- These distributions provide a measure of how extreme a *z*-score is.

 Chapter 5, Slide 17

Normal models

S_x = sample std. dev.

σ_x = population std. dev.

- There is a Normal model for every possible combination of mean and standard deviation.
  - We write $N(\overset{\bar{x}}{\mu},\sigma)$ to represent a Normal model with a mean of μ and a standard deviation of σ.
- We use Greek letters because *this* mean and standard deviation are not numerical summaries of the data. They are part of the model. They don't come from the data. They are numbers that we choose to help specify the model.
- Such numbers are called parameters of the model.

Calculating z-scores

- Summaries of data, like the sample mean and standard deviation, are written with Latin letters. Such summaries of data are called statistics.
- When we standardize Normal data, we still call the standardized value a z-score, and we write

$$z = \frac{y - \mu}{\sigma} = \frac{\text{value} - \text{mean}}{\text{S.D.}}$$

 Chapter 5, Slide 19

Practice Calculating z-scores:

$z = \frac{\text{value} - \text{mean}}{\text{S.D.}}$

1. On a math test first quarter, the mean was 86 and the standard deviation was 7, and the data fits the normal curve.

   a. If you got a 96, what was your z-score?

   $z = \frac{96-86}{7} = \frac{10}{7} = 1.43$

   b. What does this mean?

   your score was 1.43 std. dev. above the mean.

   c. Your friend got a 75, what was his z-score?

   $z = \frac{75-86}{7} = \frac{-11}{7} = -1.57$

   He scored 1.57 std. dev. below the mean.

d. What test scored has a z-score of +1.5?

$z = \frac{\text{value} - \text{mean}}{\text{S.D.}}$

$\cancel{\frac{1.5}{1} = \frac{x-86}{7}}$

$x - 86 = 10.5$
$+86 \quad +86$
$x = 96.5$

e. The teacher calls home whenever a student's z-score is worse than -2.0. What grade earns a phone call home?

$\frac{-2}{1} = \frac{x-86}{7}$

$-14 = x - 86$
$+86 \quad +86$
$72 = x$

f. In a class of 28, how many students get calls home?

$1.7 + .5 + .1 = 2.3\%$

$.023(28) = .644 \approx 1 \text{ student}$

2. 23 Kellogg's cereals were tested. Their sugar contents were measured. The mean of these cereal contents was 7.6 grams per serving with a standard deviation of 4.5 grams. Find the z-scores of the following cereals and describe what the z-score tells you about that cereal.

   a) Frosted Flakes: 11 grams per serving

$$z = \frac{11 - 7.6}{4.5} = \frac{3.4}{4.5} = .76$$

FF's amt. of sugar is .76 S.D. above the mean

   b) Apple Jacks: 14 grams of sugar per serving

$$z = \frac{14 - 7.6}{4.5} = \frac{6.4}{4.5} = 1.42$$

AJ's amt. of sugar is 1.42 S.D. above the mean

   c) Crispex: 3 grams of sugar per serving

$$z = \frac{3 - 7.6}{4.5} = \frac{-4.6}{4.5} = -1.02$$

Crispex's sugar content is 1.02 S.D. below the mean.

Homework: Normal Distribution Worksheet

Pg. 13-14
curves on pg. 15-16

For each problem, MAKE A PICTURE, MAKE A PICTURE, MAKE A PICTURE! Sketch the normal curve with data values on the curve.