Homework Answers - Day 2

1a) 50%

b) 68%

c) 2.3% or 2.5%

d) .6%

e) 66.8%

f) 15.3%

2) 4

3) 4

4) 2

5a) mean = 12.6

SD = 1.5

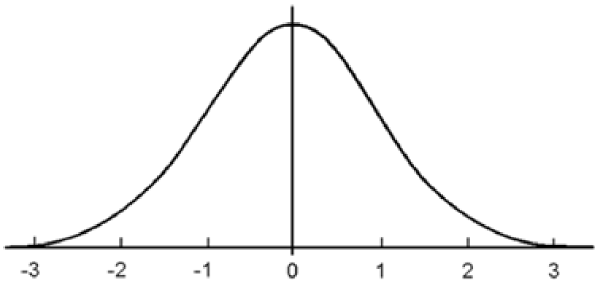
b) looks roughly symmetric with cluster of data toward the center

c) 95%

d) 6.7%

Using Z-scores to Find Percentages

Standard Normal Curve:



mean
median

If a set of data conforms to a bell-shaped (mound) curve, the data are said to be **normally distributed**. The normal curve shown above is a **Standard Normal Curve**. The standard normal curve is centered on the y-axis so that the mean is at 0 and its standard deviation is 1.

In a normal distribution, the median is the same as the mean value. So _50_ % of the data lies below the mean (0) and _50_ % of the data lies above the mean (0).

Most of the data (99.7%) is within 3 standard deviations of the mean.

When calculating probabilities associated with normal distributions use _z-scores_.

Z-scores measure _number_ of standard deviations away from the _mean_.

~~Z represents a variable that has a standard normal distribution with mean of ☐ and standard deviation of ☐.~~

Positive z-score→ corresponds to a value that's above the mean

Negative z-score→ corresponds to a value that's below the mean

memorize!

$$z = \frac{\text{value - mean}}{\text{standard deviation}}$$

1. The prices of the printers in a store have a mean of $240 and a standard deviation of $50. The printer that you eventually choose costs $340.
   a. What is the z score for the price of your printer?

$$z = \frac{340 - 240}{50} = \frac{100}{50} = 2$$

   b. How many standard deviations above the mean was the price of your printer?

2

$$z = \frac{\text{Value} - \text{Mean}}{\text{S.D.}}$$

2. Adam's height is 63 inches. The mean height for boys at his school is 68.1 inches, and the standard deviation of the boys' heights is 2.8 inches.
   a. What is the z score for Adam's height? (Round your answer to the nearest hundredth.)

$$z = \frac{63 - 68.1}{2.8} = \frac{-5.1}{2.8} = -1.82$$

   b. What is the meaning of this value?

Adam's height is 1.82 S.D.'s below the mean

3. Explain how a z score is useful in describing data.

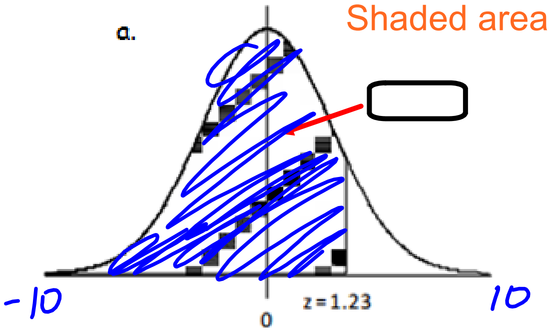
The z-score tells us how close a value is to the mean.

4. The standard normal distribution is the normal distribution with a mean of 0 and a standard deviation of 1. The diagrams below show standard normal distribution curves. Use a **table of standard normal curve areas** to determine the shaded areas. *See next slide for Z-Score Table

Shaded area = percent in the interval

Diagram in the table shows <u>shaded left</u>, so area in table is always to the <u>left</u>.

a.

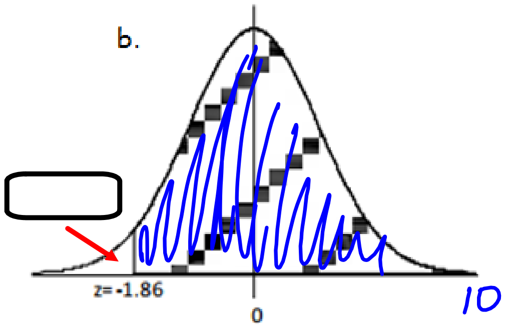


2^{nd} Vars

2: normal cdf(-10, 1.23)

.8907 ~ 89.1%

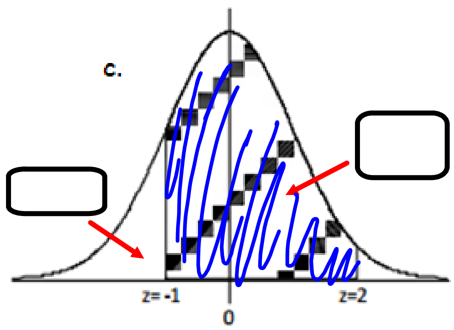
b.



normalcdf(-1.86, 10)

.9686 ~ 96.9%

c.



normalcdf(-1, 2)

.8186 ~ 81.9%

To find the probability on the normal curve between two z-scores we can use the Graphing Calculator's Normal Cumulative Density Function (Normalcdf):

Normalcdf([left/lower z bound], [right/upper z bound]) ↓ 2nd→VARS→2:normalcdf(

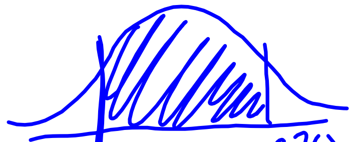
*ALWAYS DRAW A PICTURE!!!

5. A swimmer named Amy specializes in the 50 meter backstroke. In competition her mean time for the event is 39.7 seconds, and the standard deviation of her times is 2.3 seconds. Assume that Amy's times are approximately normally distributed.

   a. Using z scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is between 37 and 44 seconds.

$$z = \frac{37 - 39.7}{2.3} = -1.174$$

$$z = \frac{44 - 39.7}{2.3} = 1.870$$



-1.174 1.870

84.9%

normalcdf$(-1.174, 1.870) = .849$

   b. What is the probability that Amy's time would be at least 45 seconds?

   c. Using z scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is less than 36 seconds.

6. The distribution of lifetimes of a particular brand of car tires has a mean of 51,200 miles and a standard deviation of 8,200 miles.
   a. Assuming that the distribution of lifetimes is approximately normally distributed and rounding your answers to the nearest thousandth, find the probability that a randomly selected tire lasts
      i. between 55,000 and 65,000 miles.

      ii. less than 48,000 miles.

      iii. at least 41,000 miles.

   b. Explain the meaning of the probability that you found in a part a-iii.

   c. What's the probability that the lifetime of a randomly selected tire is within 10,000 miles of the mean lifetime for tires of this brand?

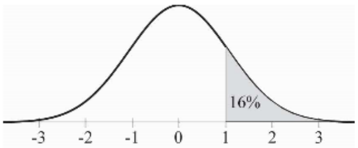
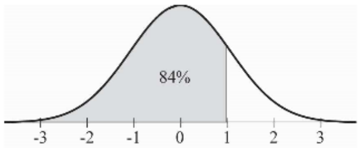
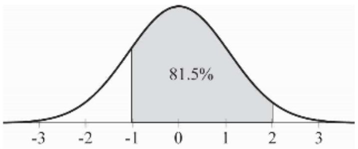
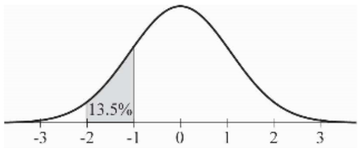
Homework:

Statistics Chapter 3: Make a Picture

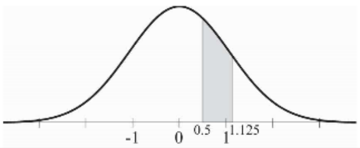
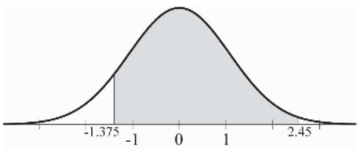
Homework Day 3 Answers:

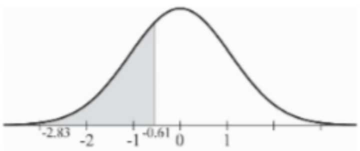
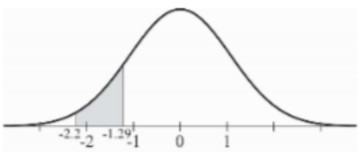
Statistics Chapter 5: Make A Picture – KEY

Practice sketching a Normal model. For each example below, mark the mean, plus and minus three standard deviations, then shade the area under of the curve as described. (Hint: Start with the mean, then use the inflection points to estimate the locations of $\pm 1\sigma$.)

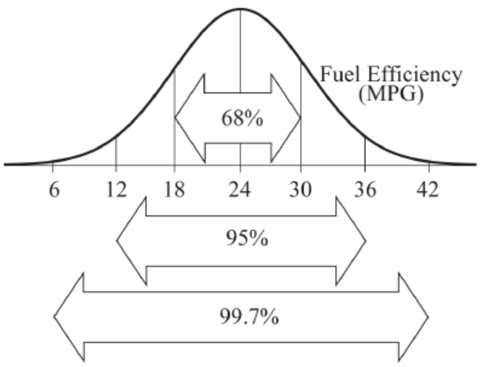
1. Greater than 1 σ above the mean 	2. Less than 1 σ above the mean 
3. Between 1 σ below the mean and 2 σ above the mean 	4. Between 1 σ below the mean and 2 σ below the mean 

5. Use the 68-95-98.7 rule to find the percent shaded in questions 1-3 above.
6. Calculate the z-scores and sketch the picture for each problem below. The first one is done for you.

Description	z calculations	Picture
a. Data between 2.7 and 3.2 in a Normal model with a mean of 2.3 and a standard deviation of 0.8.	$z = \frac{2.7-2.3}{0.8} = 0.5$ $z = \frac{3.2-2.3}{0.8} = 1.125$	
b. Data between 144 and 165.4 in a Normal model with a mean of 151.7 and a standard deviation of 5.6.	$z = \frac{144-151.7}{5.6} = -1.375$ $z = \frac{165.4-151.7}{5.6} = 2.45$	

Description	z calculations	Picture
c. Data between 5 and 52 in a Normal model with a mean of 65 and a standard deviation of 21.2.	$z = \frac{5-65}{21.2} = -2.83$ $z = \frac{52-65}{21.2} = -0.61$	
d. Between 900 and 1000 in N(1142, 110)	$z = \frac{900-1142}{110} = -2.2$ $z = \frac{1000-1142}{110} = -1.29$	

Suppose a Normal model describes the fuel efficiency of cars currently registered in your state. The mean is 24 mpg, with a standard deviation of 6 mpg.



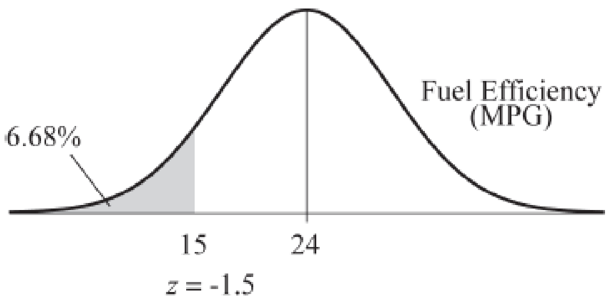
According to the Normal model, we expect 68% of cars to get between 18 and 30 mpg, 95% of cars to get between 12 and 36 mpg, and 99.7% of cars to get between 6 and 42 mpg.

What percent of all cars get less than 15 mpg?

$$z = \frac{y - \mu}{\sigma}$$

$$z = \frac{15 - 24}{6}$$

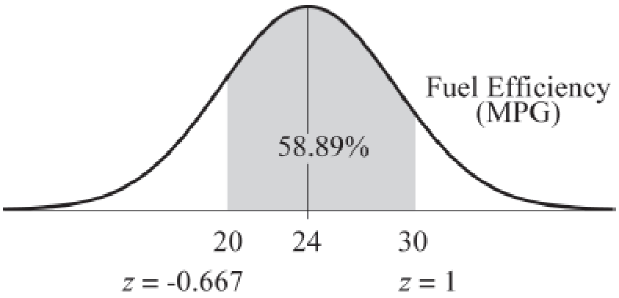
$$z = -1.5$$



According to the Normal model, about 6.68% of cars are expected to get less than 15 mpg.

What percent of all cars get between 20 and 30 mpg?

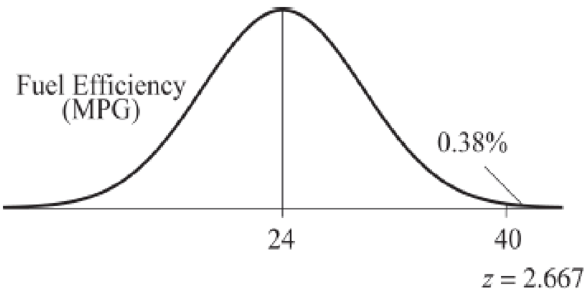
$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{20 - 24}{6}$$
$$z \approx -0.667$$



$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{30 - 24}{6}$$
$$z = 1$$

According to the Normal model, about 58.89% of cars are expected to get between 20 and 30 mpg.

What percent of cars get more than 40 mpg?



$$z = \frac{y - \mu}{\sigma}$$

$$z = \frac{40 - 24}{6}$$

$$z \approx 2.667$$

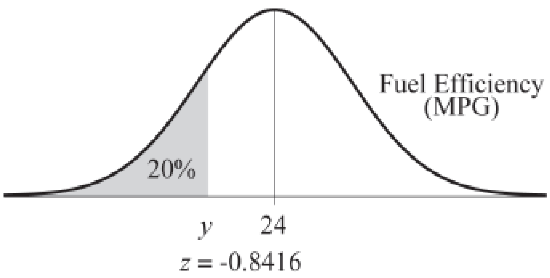
According to the Normal model, only about 0.38% of cars are expected to get more than 40 mpg.

Describe the fuel efficiency of the worst 20% of all cars.

$$z = \frac{y - \mu}{\sigma}$$

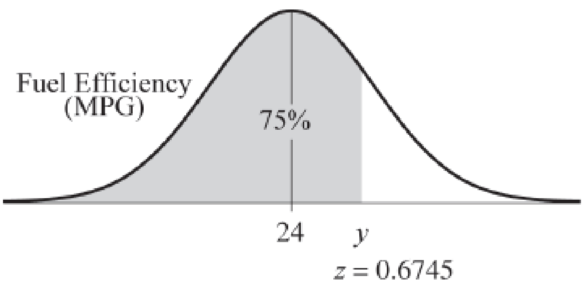
$$-0.8416 = \frac{y - 24}{6}$$

$$y \approx 18.95$$



According to the Normal model, the worst 20% of cars have fuel efficiency less than about 18.95 mpg.

What gas mileage represents the third quartile?



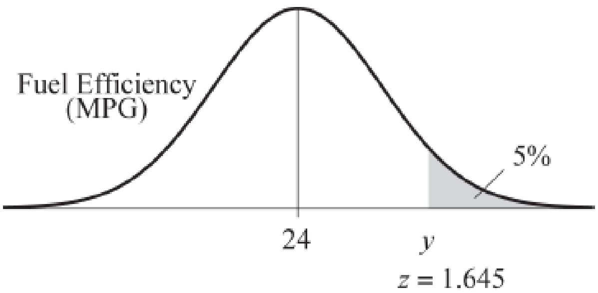
$$z = \frac{y - \mu}{\sigma}$$

$$0.6745 = \frac{y - 24}{6}$$

$$y \approx 28.05$$

According to the Normal model, the third quartile of the distribution of fuel efficiency is about 28.05 mpg.

Describe the gas mileage of the most efficient 5% of all cars.



$$z = \frac{y - \mu}{\sigma}$$

$$1.645 = \frac{y - 24}{6}$$

$$y \approx 33.87$$

According to the Normal model, we can expect the most efficient 5% of cars to have fuel efficiency in excess of 33.87 mpg.

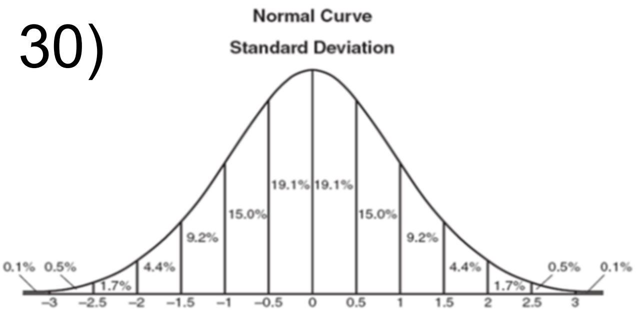
What gas mileage would you consider unusual? Why?

Answer: It depends on your definition of unusual. If ± 2 standard deviations is considered unusual, then, according to the Normal model, any gas mileage below 12 mpg or above 36 mpg would be considered unusual. If ± 3 standard deviations is considered unusual, then, according to the Normal model, any gas mileage below 6 mpg or above 42 mpg would be considered unusual.

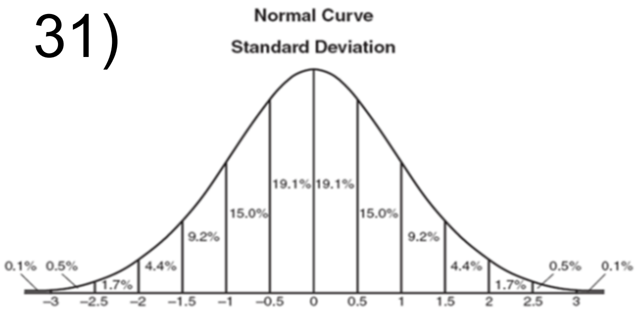
Homework:

Pg. 132-133 #30, 31, 32, 33

30)



31)



32)

33)