

Unit 2

Rational Expressions & Equations

You will learn to:

- Simplify rational expressions
- Add and subtract rational expressions
- Solve rational equations
- Multiply and divide rational expressions
- Simplify complex fractions

Rational Expressions

A rational expression is an expression that can be written as $\frac{P}{Q}$, $Q \neq 0$

where P and Q are polynomials and Q is not equal to zero.

Example: $\frac{x^2 - 4x}{x + 1} = \frac{x(x-4)}{x+1}$, $x \neq -1$

Handwritten notes: $x+1 \div 0$, -1 , -1 , $x \div -1$

When identifying values for which a rational expression is undefined, identify the values of the variable that make the original denominator = 0.

Identify any x-values for which the expression is undefined:

1. $\frac{4}{x-3}, x \neq 3$

$$x-3=0$$

$$x=3$$

3. $\frac{6x}{3x-1}, x \neq \frac{1}{3}$

$$3x-1=0$$

$$3x=1$$

$$x=\frac{1}{3}$$

2. $\frac{5}{x+4}$

4. $\frac{2}{x^2-9}, x \neq \pm 3$

$$x^2-9=0$$

$$(x-3)(x+3)=0$$

$$\begin{array}{c|c} x-3=0 & x+3=0 \\ x=3 & x=-3 \end{array}$$

Simplifying Rational Expressions

- A. Completely factor numerator and denominator, check for undefined values
 B. Divide common terms
 C. Look for a factor of -1 (ie: $\frac{5-3}{3-5} = -1$ so.... $\frac{a-b}{b-a} = -1$)

Simplify. Identify any values for which the expression is undefined:

5. $\frac{10x^4}{20x^2} : \frac{1x^2}{2}, x \neq 0$

$20x^2 : 0$
 $x : 0$

7. $\frac{x^2 - 4x}{x^2 - 2x - 8} : \frac{x(x-4)}{(x-4)(x+2)}$

$x \neq 4, -2$
 $x-4 : 0 \quad x+2 : 0 \quad : \frac{x}{x+2}, x \neq 4, -2$

6. $\frac{x+3}{x^2-2x-15} = \frac{\cancel{(x+3)}}{(x-5)\cancel{(x+3)}} = \frac{1}{(x-5)}$
 $x \neq 5, -3$

8. $\frac{5x^2 - 15x}{27x - 3x^3} = \frac{5x(x-3)}{3x(9-x^2)}$
 $= \frac{-5x(x-3)}{3x(3-x)(3+x)}$
 $x \neq 0, \pm 3$

$= \frac{-5x}{3x(3+x)} = -\frac{5}{3(3+x)}$

9. $\frac{2x^2 - 50}{x^2 + 8x + 15} = \frac{2(x^2 - 25)}{(x+5)(x+3)}$
 $x \neq -5, -3$
 $= \frac{2(x-5)(x+5)}{(x+5)(x+3)}$
 $= \frac{2(x-5)}{(x+3)}$

10. $\frac{10 - 5x}{15x - 30} = \frac{5(2-x)}{15(x-2)} = -\frac{1}{3}$
 $x \neq 2$

Determine whether or not the rational expressions $\frac{x+4}{(x+2)(x-3)}$ and $\frac{x^2+5x+4}{(x+1)(x+2)(x-3)}$ are equivalent for $x=-1$, $x=-2$ and $x=3$. **Explain your results.**

Adding or Subtracting With Like Denominators:

- A. Add the numerators - Keep the denominators
- B. Factor if possible
- C. Simplify if possible

Check for undefined

Examples: $\frac{3}{10} + \frac{6}{10} = \frac{9}{10}$ so... $\frac{3x}{10} + \frac{6x}{10} = \frac{9x}{10}$

Add or Subtract:

1. $\frac{9y}{2x} - \frac{5y}{2x} = \frac{4y}{2x} = \frac{2y}{x}$
 $x \neq 0$

2. $\frac{3x-4}{x+3} + \frac{2x+5}{x+3} = \frac{3x-4+2x+5}{x+3}$
 $x \neq -3$
 $= \frac{5x+1}{x+3}$

3. $\frac{x^2+2x}{x+1} + \frac{1}{x+1} = \frac{x^2+2x+1}{x+1}$
 $x \neq -1$
 $= \frac{(x+1)(x+1)}{(x+1)}$
 $= (x+1)$

4. $\frac{9x-2}{3x-5} - \frac{3x+8}{3x-5} = \frac{9x-2-(3x+8)}{3x-5}$
 $x \neq \frac{5}{3}$
 $= \frac{9x-2-3x-8}{3x-5}$
 $= \frac{6x-10}{3x-5}$
 $= \frac{2(3x-5)}{(3x-5)}$
 $= \underline{2}$

Discuss with your partner:

- a) How do you reduce a rational expression to lowest terms?
- b) How do you know which values of the variable(s) to exclude for a rational expression?