The number i — Algebra 2 Unit 4 Day 1

On your own, solve each equation for x.

1. $x - 1 = 0$
   $x = 1$
2. $x + 1 = 0$
   $x = -1$
3. $x^2 - 1 = 0$
   $\sqrt{x^2} = \sqrt{1}$ $x = \pm 1$
4. $x^2 + 1 = 0$
   $\sqrt{x^2} = \sqrt{-1}$ $x = \pm\sqrt{-1}$
5. $x^2 + 2 = 0$
   $\sqrt{x^2} = \sqrt{-2}$ $x = \pm\sqrt{-2}$

Which ones above do not have a real number solution? Why?

4 & 5 There is no real number that is the square root of a negative number.

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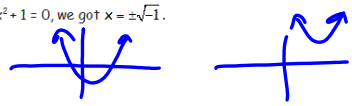
In fact, solving the equation $x^2 + 1 = 0$, we got $x = \pm\sqrt{-1}$.

This leads to $i = \sqrt{-1}$.



Problem: There is no real number that is the square root of a negative real number.

Solution: The number i.

We let $\sqrt{-1} = i$, then $i^2 = -1$.

$(\sqrt{-1})^2 = i^2$

If $r > 0$, $\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$

$r = 5$

Definition: A pure imaginary number is a number that can be written in the form bi where b is a real number and $i = \sqrt{-1}$.

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Rules of i:

1. Change all expressions of the form $\sqrt{-b}$ to $i\sqrt{b}$ first
2. Treat i as a variable for addition and subtraction.
3. Substitute -1 for i^2

Simplify:

1. $\sqrt{-9} = \sqrt{-1}\cdot\sqrt{9} = 3i$
2. $-\sqrt{-100} = -\sqrt{-1}\sqrt{100} = -10i$
3. $\sqrt{-20} = \sqrt{-1}\sqrt{4}\sqrt{5} = 2i\sqrt{5}$
4. $2\sqrt{-27} = 2\sqrt{-1}\sqrt{9}\sqrt{3}$
   $= 6i\sqrt{3}$

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Note: In the real number system $\sqrt{a}\cdot\sqrt{b} = \sqrt{a\cdot b}$. However, this is not the case when working with imaginary numbers.

Example: Simplify the following using: a)rules for real numbers, and then b)rules for i.

a) $\sqrt{-4}\cdot\sqrt{-25}$
$= \sqrt{100}$
$= 10$

b) $\sqrt{-4}\cdot\sqrt{-25}$
$= \sqrt{-1}\sqrt{4}\cdot\sqrt{-1}\sqrt{25}$
$= 2i \cdot 5i = 10i^2 = 10(-1)$
$= -10$

What do you notice?

Simplify:

5. $\sqrt{-9}\cdot\sqrt{-16} = \sqrt{-1}\sqrt{9}\cdot\sqrt{-1}\sqrt{16} = 3i\cdot 4i = 12i^2 = 12(-1) = -12$
6. $\sqrt{5}\cdot\sqrt{-10} = \sqrt{5}\cdot i\sqrt{10} = i\sqrt{50} = i\sqrt{25}\sqrt{2} = 5i\sqrt{2}$
7. $-\sqrt{-6}\cdot\sqrt{15} = -i\sqrt{6}\cdot\sqrt{15} = -i\sqrt{90} = -i\sqrt{9}\sqrt{10} = -3i\sqrt{10}$
8. $(\sqrt{-7})^2 = -7$

~~-9~~ $9i$ ~~$\sqrt{5}\,i$~~ $i\sqrt{5}$

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Put everything away.

What are some things you learned about the number i today?

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$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm\sqrt{-5}$

$x = \pm i\sqrt{5}$

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