HW 5 - 8

1. increasing: $(-\infty, 1)$
   decreasing: $(1, \infty)$
   rel min: none
   rel max: $(1, 3)$

2. increasing: $(-3, 1)$
   decreasing: $(-\infty, -3), (1, \infty)$
   rel min: $(-3, -4)$
   rel max: $(1, 4)$

3. look left to right where you would "climb the hill", graph goes higher

4. a point on the graph higher that those on either side of it

5. determine if the leading coefficient of the polynomial is + or – and decide if the degree is odd or even

Oct 29-8:36 PM

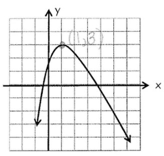
6. Graph see next page
   increasing: $(-1.44, 0), (.69, \infty)$
   decreasing: $(-\infty, -1.44), (0, .69)$
   rel min: $(-1.44, -2.83), (.69, -.40)$
   rel max: $(0, 0)$

7. Graph see next page
   increasing: $(-1.79, 1.12)$
   decreasing: $(-\infty, -1.79), (1.12, \infty)$
   rel min: $(-1.79, -8.21)$
   rel max: $(1.12, 4.06)$

Nov 1-8:32 PM

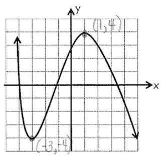
For each of the following, determine the intervals on which the graph is increasing and decreasing. Find all relative minima and maxima.

1.




Increasing: $(-\infty, 1)$
Decreasing: $(1, \infty)$
Rel Min: none
Rel Max: $(1, 3)$

2.




Increasing: $(-3, 1)$
Decreasing: $(-\infty, -3), (1, \infty)$
Rel Min: $(-3, -4)$
Rel Max: $(1, 4)$

3. How do you determine where a graph is increasing?
look left to right where you would "climb the hill" - graph goes higher

4. In your own words, what is a relative minimum?
a place on the graph higher than those on either side of it

5. How do you determine the end behavior of the graph of a polynomial function?
Determine if the leading coefficient of the polynomial is + or – and decide if the degree is odd or even

Oct 29-8:41 PM

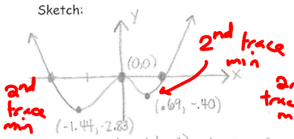
In 6 & 7, state the degree of the polynomial, find the zeros of each polynomial, state the multiplicity of each. Sketch. Using your calculator, determine relative min/max and where it's increasing/decreasing.

6. $P(x) = x^2(x + 2)(x - 1)$

Degree: 4

| Z | M | T/C |
|---|---|---|
| -2 | 1 | C |
| 0 | 2 | T |
| 1 | 1 | C |

Sketch:



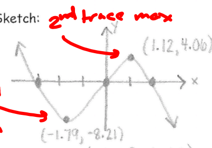

Increasing: $(-1.44, 0), (.69, \infty)$
Decreasing: $(-\infty, -1.44), (0, .69)$
Rel Min: $(-1.44, -2.83), (.69, -.40)$
Rel Max: $(0, 0)$

7. $Q(x) = -x(x + 3)(x - 2)$

Degree: 3

| Z | M | T/C |
|---|---|---|
| -3 | 1 | C |
| 0 | 1 | C |
| 2 | 1 | C |

Sketch: 2nd trace max




Increasing: $(-1.79, 1.12)$
Decreasing: $(-\infty, -1.79), (1.12, \infty)$
Rel Min: $(-1.79, -8.21)$
Rel Max: $(1.12, 4.06)$

Oct 29-8:41 PM

Warm-Up: Each of you will be assigned one of these three problems.

Remember: $F(x) \div G(x) = Q(x)$ with a remainder of $R(x)$

Which is easier to read as: *work space below - use for your problem...*

Dividend → $F(x) = G(x) \cdot Q(x) + R(x)$

Divisor Quotient Remainder

Oct 29-8:44 PM

1. Consider the polynomial function $f(x) = 3x^2 + 8x - 4$

a. Using long division, divide $f(x)$ by $x - 2$

$Q(x) =$

$R(x) =$

So $f(x) =$

b. Find $f(2)$

Oct 29-8:45 PM

2. Consider the polynomial function $g(x) = x^3 - 3x^2 + 6x + 8$

a. Using long division, divide $g(x)$ by $x + 1$

b. Find $g(-1)$

$Q(x) =$

$R(x) =$

So $g(x) =$

Oct 29-8:46 PM

3. Consider the polynomial function $h(x) = x^3 + 0x^2 + 2x - 3$

a. Using long division, divide $h(x)$ by $x - 3$

b. Find $h(3)$

$Q(x) =$

$R(x) =$

So $h(x) =$

Oct 29-8:46 PM

1. Consider the polynomial function $f(x) = 3x^2 + 8x - 4$

a. Using long division, divide $f(x)$ by $x - 2$

b. Find $f(2)$

$Q(x) = 3x+14$

$R(x) = 24$

So $f(x) = (x-2)(3x+14)+24$

$$
\begin{array}{r}
3x+14 \\
x-2\,\overline{)\,3x^2+8x-4} \\
-3x^2+6x \\
\hline
14x-4 \\
-14x+28 \\
\hline
24
\end{array}
$$

Aside:

$3x(x-2) = 3x^2 - 6x$

$14(x-2) = 14x - 28$

$f(2) = 3(2)^2 + 8(2) - 4$

$= 12+16-4$

$= 24$

Oct 29-8:45 PM

2. Consider the polynomial function $g(x) = x^3 - 3x^2 + 6x + 8$

a. Using long division, divide $g(x)$ by $x + 1$

b. Find $g(-1)$

$Q(x) = x^2-4x+10$

$R(x) = -2$

So $g(x) = (x+1)(x^2-4x+10)-2$

$$
\begin{array}{r}
x^2-4x+10 \\
x+1\,\overline{)\,x^3-3x^2+6x+8} \\
-x^3-x^2 \\
\hline
-4x^2+6x \\
4x^2+4x \\
\hline
10x+8 \\
-10x-10 \\
\hline
-2
\end{array}
$$

Aside:

$x^2(x+1) = x^3+x^2$

$-4x(x+1) = -4x^2-4x$

$10(x+1) = 10x+10$

$g(-1) = (-1)^3 - 3(-1)^2 + 6(-1) + 8$

$g(-1) = -1-3-6+8$

$g(-1) = -2$

Oct 29-8:46 PM

3. Consider the polynomial function $h(x) = x^3 + 0x^2 + 2x - 3$

a. Using long division, divide $h(x)$ by $x - 3$

b. Find $h(3)$

$Q(x) = x^2+3x+11$

$R(x) = 30$

So $h(x) = (x-3)(x^2+3x+11)+30$

$$
\begin{array}{r}
x^2+3x+11 \\
x-3\,\overline{)\,x^3+0x^2+2x-3} \\
-x^3+3x^2 \\
\hline
3x^2+2x \\
-3x^2+9x \\
\hline
11x-3 \\
-11x+33 \\
\hline
30
\end{array}
$$

Aside:

$x^2(x-3) = x^3-3x^2$

$3x(x-3) = 3x^2-9x$

$11(x-3) = 11x-33$

$h(3) = (3)^3 + 2(3) - 3$

$h(3) = 27+6-3$

$h(3) = 30$

Oct 29-8:46 PM

Write in the answers for all parts, gathered from the class discussion.
What pattern do you see?

What can we say about the connection between dividing a polynomial, P, by x – a and the value of P(a)?

Oct 29-8:47 PM

Write in the answers for all parts, gathered from the class discussion. What pattern do you see? (Answers will be posted - get them from your teachers website)

The remainder is the same as the function value of the possible zero.

Look at #1: Possible zero: 2

$R(2) = f(2)$

What can we say about the connection between dividing a polynomial, P, by x - a and the value of P(a)?

Remainder = P(a)

$R(x) = P(a)$

Oct 29-8:47 PM

In algebra, the **remainder theorem** is an application of polynomial long division.

The remainder of a polynomial p(x) divided by a linear divisor (x - c) is equal to p(c).

What does that mean?

Oct 29-8:47 PM

In algebra, the **remainder theorem** is an application of polynomial long division.

The remainder of a polynomial p(x) divided by a linear divisor (x - c) is equal to p(c).

What does that mean?

If you divide a polynomial P(x) by a possible factor (x - c), you will get a remainder that is equal to the function value of the corresponding possible zero

Oct 29-8:47 PM

Formally: $P(x) = q(x)(x - a) + P(a)$

**Why Is This Useful?**

Knowing that x - c is a factor is the same as knowing that c is a root (and vice versa).

The factor "x - c" and the root "c" are the same thing!

Now try these: Use the remainder theorem to determine the remainder.

1. $(-x^3 + 6x - 7) \div (x - 2)$

$x = 2$

$f(2) = -(2)^3 + 6(2) - 7$

$= -8 + 12 - 7$

$= 4 - 7 = -3$

2. $(x^3 + x^2 - 5x - 6) \div (x + 2)$

$x = -2$

$f(-2) = (-2)^3 + (-2)^2 - 5(-2) - 6$

$= -8 + 4 + 10 - 6$

$= 0$

Oct 29-8:50 PM

What do you think it means if the remainder is 0?

Oct 29-8:52 PM

What do you think it means if the remainder is 0?

The divisor is a factor of the polynomial; the corresponding x-value is a zero of the polynomial.

crosses the x-axis at that pt

Oct 29-8:52 PM

Another use for the remainder theorem: you can determine very quickly if a given binomial is a factor of a polynomial without doing long division or factoring.

Using the remainder theorem, determine if the given binomial is a factor of the given polynomial.

1. $(x^3 - x^2 - x - 2) \div (x - 2)$

$x=2$

2. $(x^4 - 8x^3 - x^2 + 62x - 34) \div (x - 7)$

$x=7$

Oct 29-8:53 PM

3. Given the polynomial $P(x) = x^3 + kx^2 + x + 6$

a. Find the value of k so that $x + 1$ is a factor of P.

$P(-1)=0$

$(-1)^3+k(-1)^2+(-1)+6=0$

$-1+k+5=0$ $\quad k+4=0$ $\quad k=-4$

b. Find the other two factors of P for the value of k found in part a.

$P(x)=x^3-4x^2+x+6$

$$
\begin{array}{r|l}
 & x^2-5x+6 \\
x+1 & x^3-4x^2+x+6 \\
x^2(x+1) & -(x^3+x^2) \\
 & -5x^2+x \\
-5x(x+1) & +5x^2+5x \\
 & 6x+6 \\
6(x+1) & -(6x+6) \\
 & 0
\end{array}
$$

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x-3=0$ | $x-2=0$

$x=3$ | $x=2$

Oct 29-8:54 PM

Nov 19-7:31 AM