Day 5: Finding Trig Values

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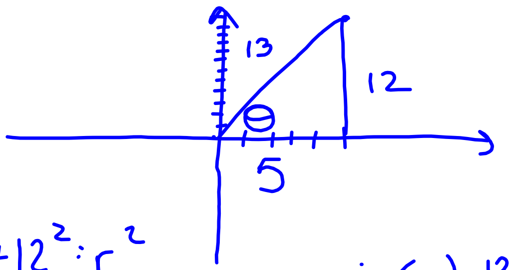
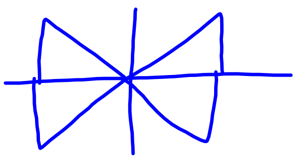
Warm-Up:
P(5, 12) is a point on the terminal side of θ in standard position. Find the exact values of sin(θ), cos(θ), and tan(θ).





"Bowtie"

$5^2 + 12^2 : r^2$

$\sin(\theta) : \frac{12}{13}$ $\quad \frac{O}{H}$

$\cos(\theta) : \frac{5}{13}$ $\quad \frac{A}{H}$

$\tan(\theta) : \frac{12}{5}$ $\quad \frac{O}{A}$

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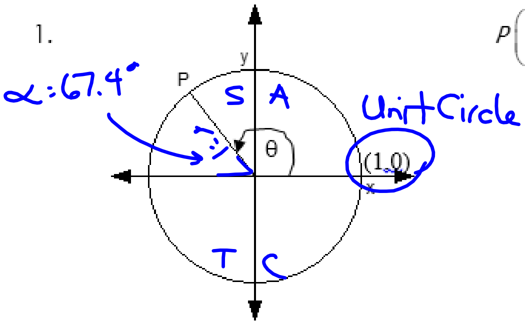
Finding Trig Values Given a Point:

* Remember that if a point is **on the unit circle**, radius = 1
the x-coordinate = cos(θ), the y-coordinate = sin(θ)
and tan(θ) = y/x = sin(θ)/cos(θ). To find ∠θ, we find the reference angle first using the positive lengths of the triangle sides, and then put that angle into the correct quadrant to find ∠θ.

Examples:

(cos(θ), sin(θ))

1. $P\left(\frac{-5}{13}, \frac{12}{13}\right)$



sin(θ) = $\frac{12}{13}$

cos(θ) = $-\frac{5}{13}$

tan(θ) = $-\frac{12}{5}$

$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{12}{13}}{\frac{-5}{13}} \qquad \frac{12}{\cancel{13}} \cdot \frac{\cancel{13}}{1} \qquad \frac{-5}{\cancel{13}} \cdot \frac{\cancel{13}}{1}$$

m ∠θ = 180° − 67.4° = 112.6°

$\alpha = \sin^{-1}\left(\frac{12}{13}\right) \approx 67.4°$

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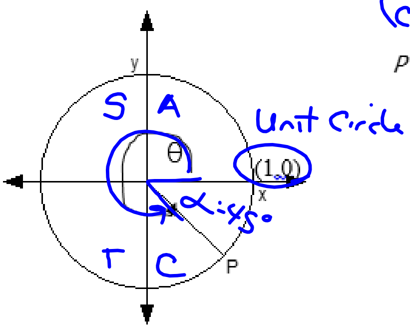
(cos(θ), sin(θ))

2. $P\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$



sin(θ) = $-\frac{\sqrt{2}}{2}$

cos(θ) = $\frac{\sqrt{2}}{2}$

tan(θ) = −1

$$\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

m∠θ = 360° − 45° = 315°

$\alpha = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$

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3. Point $A\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ is on a unit circle with a center of the origin. If θ is an angle in standard position whose terminal side passes through A, find:

$r = 1$

a. sin(θ) $: -\frac{\sqrt{3}}{2}$

b. cos(θ) $: \frac{1}{2}$

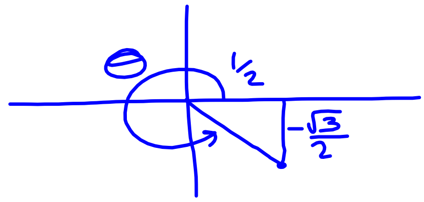
c. tan(θ) $: -\sqrt{3}$

$-\frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

$\frac{1}{2} \cdot \frac{2}{1}$

d. m∠θ $: 360° - 60° = 300°$

$\alpha : \cos^{-1}\left(\frac{1}{2}\right) : 60°$



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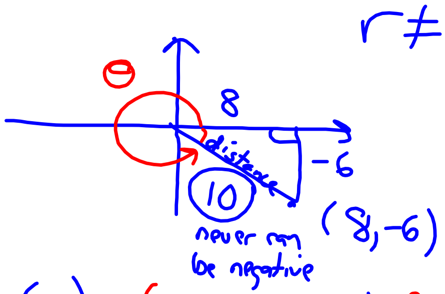
4. P(8, -6) is a point on the terminal side of θ in standard position. Find the exact values of sin(θ), cos(θ) and tan(θ).

Why is this example different? not on the unit circle

$r \neq 1$



$8^2 + (-6)^2 = r^2$ $\quad r = \sqrt{x^2 + y^2}$

$64 + 36 = r^2$ $\quad r = \sqrt{64 + 36}$

$\sqrt{r^2} = \sqrt{100}$

$r = 10$

$\sin(\theta) : \frac{-6}{10} : \frac{-3}{5}$ $\quad \cos(\theta) : \frac{8}{10} : \frac{4}{5}$ $\quad \tan(\theta) : \frac{-6}{8} : -\frac{3}{4}$

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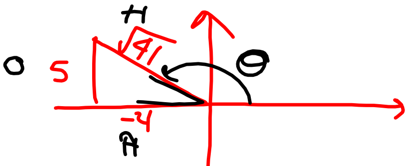
5. P(-4, 5) is a point on the terminal side of θ in standard position. Find the exact values of sin(θ), cos(θ) and tan(θ).



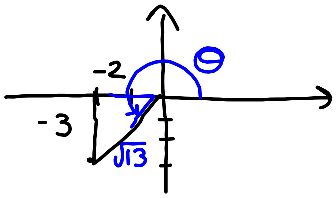
$r = \sqrt{25+16} = \sqrt{41}$

$\sin(\theta) = \frac{5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

$\cos(\theta) = \frac{-4}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \frac{-4\sqrt{41}}{41}$

$\tan(\theta) = -\frac{5}{4}$

6. P(-2, -3) is a point on the terminal side of θ in standard position. Find the exact values of sin(θ), cos(θ) and tan(θ).



$r = \sqrt{4+9} = \sqrt{13}$

$\sin(\theta) = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$

$\cos(\theta) = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

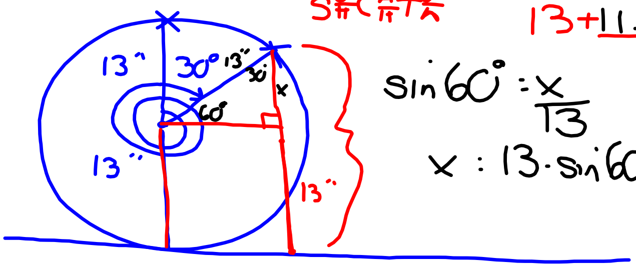
$\tan(\theta) = \frac{-3}{-2} = \frac{3}{2}$

Application Word Problems:

1. A bicycle wheel with a radius of 13" has a valve cap positioned at the highest point of the wheel. If the wheel is spun 750° in one direction, how high is the valve cap above the ground? Round your answer to the nearest tenth of an inch.

SOHCAHTOA

$13 + 11.3 = 24.3''$



$\sin 60° = \frac{x}{13}$

$x = 13 \cdot \sin 60°$

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