AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA II

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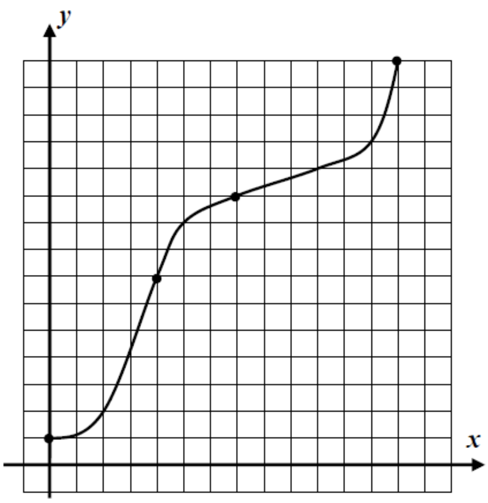
When we model using functions, we are very often interested in the rate that the output is changing compared to the rate of the input.

***Exercise* #1:** The function $f(x)$ is shown graphed to the right.



(a) Evaluate each of the following based on the graph:

(i) $f(0)$ $=1$ (ii) $f(4)$ $=7$ (iii) $f(7)$ $=10$ (iv) $f(13)$ $=15$

(b) Find the change in the function, Δf, over each of the following domain intervals. Find this both by subtraction and show this on the graph. $\Delta f = y_2 - y_1$

(i) $0 \le x \le 4$ (ii) $4 \le x \le 7$ (iii) $7 \le x \le 13$

$\Delta f = 7 - 1 = 6$ $\Delta f = 10 - 7 = 3$ $\Delta f = 15 - 10 = 5$

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(c) Why can't you simply compare the changes in f from part (b) to determine over which interval the function changing the fastest?

Because these changes don't account for the fact that the domain intervals are not the same width or size. We need to compare the change in y to the change in x.

(d) Calculate the **average rate of change** for the function over each of the intervals and determine which interval has the greatest rate.

Greatest Rate of Change

(i) $0 \le x \le 4$

$\frac{\Delta f}{\Delta x} = \frac{6}{4-0} = \frac{6}{4} = \frac{3}{2}$

(ii) $4 \le x \le 7$

$= \frac{3}{7-4} = \frac{3}{3} = 1$

(iii) $7 \le x \le 13$

$= \frac{5}{13-7} = \frac{5}{6}$

(e) Using a straightedge, draw in the lines whose slopes you found in part (d) by connecting the points shown on the graph. The average rate of change gives a measurement of what property of the line?

The average rate of change, or the slope, measures how steep the line is that connects the two points on the function.

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The average rate of change is an exceptionally important concept in mathematics because it gives us a way to **quantify** how fast a function changes on average over a certain **domain interval**. Although we used its formula in the last exercise, we state it formally here:

AVERAGE RATE OF CHANGE

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$$\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b)-f(a)}{b-a}$$

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***Exercise* #2:** Consider the two functions $f(x)=5x+7$ and $g(x)=2x^2+1$.

(a) Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

(i) $-2 \le x \le 3$

$f(-2) = 5(-2)+7 = -3$

$f(3) = 5(3)+7 = 22$

$\dfrac{f(3)-f(-2)}{3-(-2)} = \dfrac{22-(-3)}{5} = \dfrac{25}{5} = 5$

$g(-2) = 2(-2)^2+1 = 9$

$g(3) = 2(3)^2+1 = 19 \Rightarrow \dfrac{19-9}{5} = \dfrac{10}{5} = 2$

(ii) $1 \le x \le 5$

$f(1) = 5(1)+7 = 12$

$f(5) = 5(5)+7 = 32$

$= \dfrac{32-12}{5-1} = \dfrac{20}{4} = 5$

$g(1) = 2(1)^2+1 = 3$

$g(5) = 2(5)^2+1 = 51$

$= \dfrac{51-3}{5-1}$

$= \dfrac{48}{4} = 12$

(b) The average rate of change for f was the same for both (i) and (ii) but was not the same for g. Why is that?

The first function is linear. The defining characteristic of a linear function is that its average rate of change is constant. The second function is quadratic, which does not have a constant average rate of change.

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***Exercise* #3:** The table below represents a linear function. Fill in the missing entries.

x	1	5	11	19	45
y	-5	1	10	22	61

Linear Function $: \dfrac{1-(-5)}{5-1} = \dfrac{6}{4} = \dfrac{3}{2}$

Average Rate must be constant

For $x = 11$

$\dfrac{y-1}{11-5} = \dfrac{3}{2}$

$\dfrac{y-1}{6} = \dfrac{3}{2}$

$2y-2 = 18$

$2y = 20$

$y = 10$

For $y = 22$

$\dfrac{22-10}{x-11} = \dfrac{3}{2}$

$3x-33 = 24$

$\dfrac{3x}{3} = \dfrac{57}{3}$

$x = 19$

For $x = 45$

$\dfrac{y-22}{45-19} = \dfrac{3}{2}$

$\dfrac{y-22}{26} = \dfrac{3}{2}$

$2y-44 = 78$

$2y = 122$

$y = 61$

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