Average Rate of Change

HW 9-1

1) a. -5
b. 2
c. $-\frac{1}{4}$
d. See explanation

2) a. 2
b. 6
c. 10
d. See explanation

3) g(x)
See work

4) a. 22
b. $11 \le t \le 14$

5) See explanation

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Name: ______________________ Date: ______________

AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. For the function $g(x)$ given in the table below, calculate the average rate of change for each of the following intervals.

x	-3	-1	4	6	9
$g(x)$	8	-2	13	12	5

(a) $-3 \le x \le -1$

$= \frac{-2-8}{-1-(-3)} = \frac{-10}{2}$
$= -5$

(b) $-1 \le x \le 6$

$= \frac{12--2}{6--1} = \frac{14}{7}$
$= 2$

(c) $-3 \le x \le 9$

$= \frac{5-8}{9--3} = \frac{-3}{12}$
$= -\frac{1}{4}$

(d) Explain how you can tell from the answers in (a) through (c) that this is **not** a table that represents a linear function.

If this was a linear function then the average rate of change would have been the same for each of these intervals.

2. Consider the simple quadratic function $f(x) = x^2$. Calculate the average rate of change of this function over the following intervals:

(a) $0 \le x \le 2$

$= \frac{f(2)-f(0)}{2-0}$

$\frac{4-0}{2} = 2$

(b) $2 \le x \le 4$

$\frac{f(4)-f(2)}{4-2}$

$\frac{16-4}{2} = 6$

(c) $4 \le x \le 6$

$\frac{f(6)-f(4)}{6-4}$

$\frac{36-16}{2} = 10$

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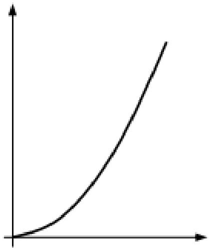
(d) Clearly the average rate of change is getting larger at x gets larger. How is this reflected in the graph of f shown sketched to the right?



As x gets larger the y-values are increasing in larger intervals.

or

The graph is getting steeper as we move from left to right.

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3. Which has a greater average rate of change over the interval $-2 \le x \le 4$, the function $g(x) = 16x - 3$ or the function $f(x) = 2x^2$? Provide justification for your answer.

$g(x)$

$= \dfrac{g(4) - g(-2)}{4 - -2}$

$= \dfrac{61 - -35}{6} = \dfrac{96}{6} = 16$

$f(x)$

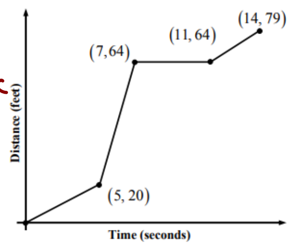
$= \dfrac{f(4) - f(-2)}{4 - -2}$

$= \dfrac{32 - 8}{6} = \dfrac{24}{6} = 4$

g(x) has a greater rate of change.

Applications

4. An object travels such that its distance, d, away from its starting point is shown as a function of time, t, in seconds, in the graph below.



(a) What is the average rate of change of d over the interval $5 \le t \le 7$? Include proper units in your answer.

$= \dfrac{64 - 20}{7 - 5} = \dfrac{44}{2} = 22$ ft/sec

(b) The average rate of change of distance over time (what you found in part (a)) is known as the **average speed** of an object. Is the average speed of this object greater on the interval $0 \le t \le 5$ or $11 \le t \le 14$? Justify.

$0 \le t \le 5$: $\dfrac{20 - 0}{5 - 0} = 4 \frac{ft}{sec}$

$11 \le t \le 14$: $\dfrac{79 - 64}{14 - 11} = 5 \frac{ft}{sec}$

The average speed is slightly greater on the interval $11 \le t \le 14$

Reasoning

5. What makes the average rate of change of a linear function different from that of any other function? What is the special name that we give to the average rate of change of a linear function?

The average rate of change is a constant for linear functions and is not dependent on the interval over which it is calculated. We call this average rate of change the slope.

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Rules of Exponents

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Rules of Exponents Unit 9 Day 2

	Rule:	Examples:
Product Rule →	$x^a \cdot x^b = x^{a+b}$	a. $x^5 \cdot x^3 = x^{5+3} = x^8$
		b. $2^x \cdot 2^3 = 2^{x+3}$
Quotient Rule →	$\frac{x^a}{x^b} = x^{a-b}$	a. $\frac{x^7}{x^2} = x^5$
		b. $\frac{4^x}{4^3} = 4^{x-3}$
Power Rule →	$(x^a)^b = x^{ab}$	a. $(x^3)^4 = x^{12}$
		b. $(3^2)^3 = 3^6$

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Power of a Product → $(xy)^a = x^a y^a$

* Work Backwards

a. $(ab)^6 = a^6 b^6$

b. $2^3 \cdot 3^3 = (2\cdot 3)^3 = 6^3$

Power of a Quotient → $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

a. $\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$

b. $\left(\frac{x^3}{4}\right)^2 = \frac{x^6}{4^2} = \frac{x^6}{16}$

Zero Exponent → $x^0 = 1$

a. $3x^0 = 3 \cdot 1 = 3$

b. $(3x)^0 = 1$

c. $-(3x)^0 = -1$

* 0^0 → undefined

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Simplify each expression:

1. $3a^2b^3c^4 \cdot 5ab^2c^6 = 15a^3b^5c^{10}$

2. $\frac{35x^4y^7z^{10}}{7xy^5z^{10}} = 5x^3y^2$

PEMDAS

3. $3(2x^3y^2)^3$

$= \underline{3 \cdot 8}x^9y^6$

$= 24x^9y^6$

4. $-2(-2x^3y)^2$

$= -2 \cdot 4x^6y^2$

$= -8x^6y^2$

5. $\frac{x^{3b}}{x^b} = x^{2b}$

x^{3b-b}

6. $y^{a+1} \cdot y^{a-1} = y^{a+1+a-1}$

$= y^{2a}$

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7. $\dfrac{5x^3y^7}{4x^2y^2}$ $= \dfrac{5xy^5}{4}$

8. $\left(\dfrac{5x^2}{2y}\right)^3 = \dfrac{5^3x^6}{2^3y^3}$

PEMDAS

$= \dfrac{125x^6}{8y^3}$

PEMDAS

9. $\left(\dfrac{-12a^8b^5}{6a^2b^4}\right)^2 = (-2a^6b)^2$

$= 4a^{12}b^2$

10. $-\dfrac{4^0}{5} = -\dfrac{1}{5}$

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Do the following without a calculator. Use the rules of exponents to help you evaluate the expression.

Express 8^3 as a power of 2. $\longrightarrow$ base has to be 2

$8 = 2^{\boxed{3}}$ $(2^3)^3 = 2^9$

Divide 4^{15} by 2^{10}. $\dfrac{4^{15}}{2^{10}} = \dfrac{(2^2)^{15}}{2^{10}} = \dfrac{2^{30}}{2^{10}} = 2^{20}$

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Using the power rule evaluate 16 times 9.

$(xy)^a$

$4^2 \cdot 3^2 = (4 \cdot 3)^2 = 12^2 = 144$

Using the power rule multiply 25 times 9.

$5^2 \cdot 3^2 = (5 \cdot 3)^2 = 15^2 = 225$

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Apply the properties of exponents to verify that each statement is an identity.

$\frac{2^{n+1}}{3^n} = 2\left(\frac{2}{3}\right)^n$

$\frac{2^{n+1}}{3^n} = 2\left(\frac{2}{3}\right)^n$

1. Product Rule
2. Power of a Quotient

$\frac{2^n \cdot 2^1}{3^n}$

$2 \cdot \frac{2^n}{3^n}$

$2 \cdot \left(\frac{2}{3}\right)^n$

gcf

product rule

$3^{n+1} - 3^n = 2(3^n)$

$3^n \cdot 3^1 - 3^n$

$3^n(3^1 - 1)$

$2 \cdot 3^n$

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