Pg 407-408

14. 1.256
20. −0.612
28. $\frac{1}{8}$
36. 3
38. 11
48. 3

Nov 30-9:21 AM

## Word Problem Mixture Classwork/Homework

$\underbrace{A = Pe^{rt} \qquad A = P\left(1+\frac{r}{n}\right)^{nt}}_{\text{money}} \qquad \underbrace{A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}}_{\text{growth/decay}}$

Nov 27-1:54 PM

Remember from last year:

There is a special base that we have to consider. What if we are asked to evaluate something that has a half-life every so many days? Now we need to have a special base of $\frac{1}{2}$. The words that follow the word **EVERY** will be what we use for the exponent. The "every whatever" will always be the denominator of the exponent. Let us fill in the box below and represent this general situation.

The same holds true for a quantity being doubled, tripled, etc. Let us fill in the box below and represent this general situation.

| Growth | Decay | Special "half-life" | Double/Triple/ etc. |
|---|---|---|---|
| $b^t$<br>$b>1$ | $b^t$<br>$0<b<1$ | $\left(\frac{1}{2}\right)^{t/\text{every}}$ | $(2)^{t/\text{every}}$<br>$(3)^{t/\text{every}}$ |

Let us represent the following situations according to what we just discovered above.

Double every 3 days $(2)^{t/3}$

Triple every 15 minutes $(3)^{t/15}$

Half every 3.7 years $\left(\frac{1}{2}\right)^{t/3.7}$

Remember that whatever follows the every is always the denominator of the exponent.

Nov 27-1:55 PM

Example: If a population of honeybees doubles every 5 years, how many years to the *nearest tenth of a year* will it take the population to increase by 10 times the original amount?

$10 = 1(2)^{t/5}$

$\log 10 = \frac{t}{5}\log 2$

$t = 5\left(\frac{\log 10}{\log 2}\right) = 16.6$ years

Nov 27-1:57 PM

$A = Pe^{rt} \qquad A = P\left(1+\frac{r}{n}\right)^{nt} \qquad A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

① Do I need an exponent? ② Ln or Log? ③ Money or Growth?

For each question, solve for the missing variable before using your calculator to evaluate.

1. Byron invests \$1000 at First Rate Savings where they offer an account with 4.4% APR compounded continuously. What would the balance in his account be after 4 years?

$A = 1000e^{.044(4)}$

$A = \$1192.44$

Nov 14-3:14 PM

2. When Angelina was born, her grandparents deposited \$3000 into a college savings account pay 4% interest compounded continuously.

a. Assuming there are no deposits or withdrawals from the account, what will the balance be after 10 years?

$A = 3000e^{.04(10)}$ $\qquad A = \$4475.47$

b. How long will it take the balance to reach at least \$10,000?

$10000 = 3000e^{.04t}$

$\frac{10}{3} = e^{.04t}$

$\ln\frac{10}{3} = .04t \ln e$

$t = \frac{\ln\frac{10}{3}}{.04}$

$t \approx 30$ years

c. If her grandparents want her to have \$10000 after 18 years, how much would they need to invest?

$10000 = Pe^{18(.04)}$

$P = \frac{10000}{e^{18(.04)}}$

$P = \$4867.52$

Nov 14-3:18 PM

3. If interest is compounded quarterly, what annual rate must you receive if your investment of $1500 is to grow to $2100 in six years? (round to the nearest hundredth)

$2100 = 1500\left(1+\frac{r}{4}\right)^{4(6)}$

$1.4^{(1/24)} = \left(1+\frac{r}{4}\right)^{24\left(\frac{1}{24}\right)}$ or $\rightarrow \sqrt[24]{1.4} = \sqrt[24]{\left(1+\frac{r}{4}\right)}$

or

$(1.4)^{1/24} - 1 = \frac{r}{4}$

$r = 4\left((1.4)^{1/24} - 1\right)$

$r = 5.65\%$

Nov 14-3:19 PM

4. Mrs. Chapman invests $8500 in a retirement account with a fixed annual interest rate of 7% compounded monthly. How long will it take for the balance to reach $70000? (round to the nearest month)

$70000 = 8500\left(1+\frac{.07}{12}\right)^{12t}$

$\log\frac{140}{17} = 12t\log\left(1+\frac{.07}{12}\right)$

$t = \dfrac{\log\frac{140}{17}}{12\log\left(1+\frac{.07}{12}\right)}$

$t =$ 30 years & 3 months

Nov 14-3:19 PM

5. What is the half-life ($h$) of a radioactive isotope if a 500.0 gram sample decays to 62.5 grams in 24.3 hours?

$62.5 = 500\left(\frac{1}{2}\right)^{24.3/h}$

$\log(.125) = \frac{24.3}{h}\log(.5)$

$h = \dfrac{24.3\log(.5)}{\log(.125)}$

$h = 8.1$ hours

Nov 14-3:19 PM

6. A super-deadly strain of bacteria is causing the zombie population to double every 2 days. Currently, there are 25 zombies. After how many days will there be 25,600 zombies?

$25600 = 25(2)^{t/2}$

$1024 = 2^{t/2}$

$2^{10} = 2^{t/2}$

$10 = \frac{t}{2}$

$t = 20$ days

Nov 14-3:19 PM

7. ~~[illegible] exponentially at a school. The number of new fl[illegible] can be modeled using the equation [illegible] the number of days since 10 students had the flu. [illegible] days will it take for the [illegible] new flu patients to [illegible]~~

$50 = 10(2)^{t/6}$

$\log 5 = \frac{t}{6}\log(2)$

$t = \frac{\log(5)}{\log(2)} \cdot 6 \approx 13.93 \approx 14$ days

Nov 14-3:20 PM

8. Mr. Cuthbert deposits $5000 into an account paying 4.3% interest compounded semi-annually. How long until he has $8000 in the account?

$8000 = 5000\left(1+\frac{.043}{2}\right)^{2t}$

$\log(1.6) = 2t\log(1.0125)$

$t = \dfrac{\log(1.6)}{2\log(1.0125)}$

$t \approx 11$ yrs

Nov 14-3:20 PM

9. Carbon-14 has been used to date the La Brea Tar Pits by testing remains of Saber Tooth Tigers. Carbon-14 has a half-life of 5,730 years. Scientifically it is assumed that the initial mass of Carbon-14 abundant in a saber tooth femur was 1 microgram. If the measurements indicate that the current mass of Carbon-14 in the femur is approximately 0.09 micrograms, how many years ago did the saber tooth die? (round to the nearest 10 years)

$$.09 = 1\left(\frac{1}{2}\right)^{t/5730}$$

$$\log .09 = \frac{t}{5730}\log(.5)$$

$$t = \frac{5730\log .09}{(\log .5)}$$

$t \approx 19910$ years ago

Nov 14-3:20 PM

10. Mrs. Phelps has $4000 she is going to invest in an account that compounds continuously. She plans to keep her money in the account for 15 years and would like it to be worth $7500 when she takes it out. What interest rate will she need to receive? (round to the nearest tenth)

$$7500 = 4000e^{15r}$$

$$1.875 = e^{15r}$$

$$\ln 1.875 = 15r\,\cancel{\ln e}$$

$$r = \frac{\ln 1.875}{15}$$

$r = 4.2\%$

Nov 14-3:20 PM