P. 388

51) $a^{-x} = M$

PP. 397-398

4) $\log_4 64 + \log_4 32$

11) 8log y

19) log x – log y

24) $3\log_a x + 2\log_a y + \log_a z$

26) $2\log_b x + \log_b y - 3\log_b b$

29) $\frac{3}{2}\log r + \frac{1}{2}\log t$

39) $\log(m^3\sqrt{n})$

46) $\log(x^2 + 2x + 4)$

55) 1.991

$\log_a 98$

$\log_a 2 \cdot 7 \cdot 7$

$\log_a 2 + \log_a 7 + \log_a 7$

$\_ + \_ + \_$:

Nov 15-2:56 PM

Applications of Logs

Nov 10-11:11 AM

Compounded Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

P = principal (amount invested)
A = Amount after t years
e = Euler's number

Compounded Continuously: $A = Pe^{rt}$

r = rate as a decimal
t = number of years
n = number of compounds per year

Nov 10-9:00 AM

1. If $5000 is invested at 6.5% annual interest rate, how much will you have after $3\frac{1}{2}$ years if the money is compounded:

a. Annually $n = 1$
$A: 5000(1.065)^{3.5}$
$: \$6232.95$

b. Quarterly $n = 4$ Alpha y:
$A: 5000\left(1 + \frac{.065}{4}\right)^{4(3.5)}$
$A: \$6265.82$

c. Monthly $n = 12$
$A: 5000\left(1 + \frac{.065}{12}\right)^{12(3.5)}$
$A: \$6273.43$

d. Continuously $A: Pe^{rt}$
$A = 5000e^{.065(3.5)}$
$A: \$6277.29$

Nov 10-9:02 AM

2. How long will it take your $5000 invested at 6.5% to double compounded:

$t = ?$

a. Monthly $n = 12$

$\frac{10000}{5000} = \frac{5000\left(1 + \frac{.065}{12}\right)^{12t}}{5000}$

$2 = \left(1 + \frac{.065}{12}\right)^{12t}$

$\log 2 = \log\left(1 + \frac{.065}{12}\right)^{12t}$

$\log 2 = 12t\left(\log\left(1 + \frac{.065}{12}\right)\right)$

$\frac{\log 2}{12\log\left(1 + \frac{.065}{12}\right)} = \frac{12\log\left(1 + \frac{.065}{12}\right)}{12\log\left(1 + \frac{.065}{12}\right)}$

$t = 10.7$ years

b. Continuously

$A: Pe^{rt}$

Alpha y:

$10000 : 5000e^{.065t}$

$2 = e^{.065t}$

$\ln 2 = .065t \ln e$

$\frac{\ln 2}{.065} = \frac{.065t}{.065}$

$t: 10.7$ years

Nov 28-10:30 AM

3. If you put $5000 in an account that pays interest quarterly, what interest rate must you receive in order to have $7500 after 5 years?

$r = ?$

$7500 = 5000\left(1 + \frac{r}{4}\right)^{4.5}$

$\sqrt[20]{\frac{7500}{5000}} = \sqrt[20]{\left(1 + \frac{r}{4}\right)^{20}}$

$\sqrt[20]{\frac{75}{50}} = 1 + \frac{r}{4}$

$1.02 = 1 + \frac{r}{4}$

$-1 \quad -1$

$4(.02) = \left(\frac{r}{4}\right)4$

$r = .0819$

8.2%

Nov 10-9:03 AM

4. Mike invests $6000 in a retirement account with a fixed annual interest rated compounded continuously. After 15 years, his balance is $8099.15. What is the interest rate on the account?

$A = Pe^{rt}$

$\frac{8099.15}{6000} = \frac{6000e^{15r}}{6000}$

$\frac{\ln\left(\frac{8099.15}{6000}\right)}{15} = \frac{15r \ln e}{15}$

$r = .0199$ 2%

Nov 10-9:03 AM

The mass of a radioactive element at time t is given by

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Where A_0 is the initial mass and h is the half-life of the element.

5. After 43 years, a 20-milligram sample of strontium-90 (^{90}Sr) decays to 6.071 mg. What is the half-life of strontium-90? h: ?

$\frac{6.071}{20} = \frac{20\left(\frac{1}{2}\right)^{43/h}}{20}$

$\frac{6.071}{20} = \left(\frac{1}{2}\right)^{43/h}$

$\frac{\log\left(\frac{6.071}{20}\right)}{\log\left(\frac{1}{2}\right)} = \frac{\frac{43}{h}\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$ h: 25 years

$1.72 = \frac{43}{h}$

$\frac{1.72h}{1.72} = \frac{43}{1.72}$ 43 ÷ 2nd answer

Nov 10-9:03 AM

6. When a living organism dies, its carbon-14 decays. The half-life of carbon-14 is 5730 years. If the skeleton of a mastodon has lost 58% of its original carbon-14, when did the mastodon die? (to the nearest hundred years) t: ?

$(1-.58) = .42 = 1\left(\frac{1}{2}\right)^{t/5730}$

$\frac{\log(.42)}{\log\left(\frac{1}{2}\right)} = \frac{\frac{t}{5730}\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$

$5730 \cdot \left(\frac{\log(.42)}{\log\left(\frac{1}{2}\right)}\right) = t$

$t = 7171.3 \approx 7200$ years ago

Nov 10-9:03 AM

Nov 27-7:20 AM